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MODUL BELGISI QATNASHGAN TRIGONOMETRIK TENGLAMALARNI YECHISH BO‘YICHA MULOHAZALAR

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Maqola umumiy o‘rta ta’lim maktablarida o‘qitiladigan va o‘quvchilar o‘zlashtirishda biroz qiyinchiliklarga duch keladigan trigonometrik tenglamalarni o‘rganishga bag‘ishlangan bo‘lib, unda modul belgisi qatnashgan trigonometrik tenglamalar va ularning yechish usullari bayon qilingan. Shuningdek, maqolada haqiqiy sonning moduli va uning xossalari misollar yordamida yoritib berilgan. Modul qatnashgan trigonometrik tenglamalarga doir misollar ham yechimlari bilan keltirilgan.

Kalit so‘zlar: *haqiqiy sonning moduli, modulli tenglama, trigonometrik tenglama, umumiy yechim, modulli ifoda, yig‘indining moduli, ko‘paytmaning moduli, bo‘linmaning moduli.*

Статья посвящена изучению тригонометрических уравнений, которые считаются одной из трудноусваиваемых тем по математике в средней общеобразовательной школе. В ней приведены тригонометрические уравнения, содержашие знак модуля. Кроме того, рассматриваются определение абсолютного значения действительного числа и его основные свойства, которые часто применяются при решении тригонометрических уравнений с модулем. Также в статье приведены примеры тригонометрических уравнений с решениями.

Ключевые слова: *абсолютная величина действительного числа, уравнение, содержащее знак модуля, тригонометрическое уравнение, общее решение, выражение, содержащее знак модуля, абсолютная величина суммы, абсолютная величина произведения, абсолютная величина частного.*

Ma’lumki, so‘nggi yillarda Respublikamizda ta’lim tizimini, jumladan, matematik ta’limni tubdan isloh qilishga katta ahamiyat berilmoqda. Bunga yaqqol misol sifatida Respublikamizda matematika ta’limi va fanlarini yanada rivojlantirish chora-tadbirlari to‘g‘risidagi Prezident qarorini keltirish mumkin. Bu qarorda qo‘yilgan vazifalar uzluksiz ta’lim tizimining muhim bo‘g‘inlaridan hisoblangan umumiy o‘rta ta’lim maktablari bo‘g‘iniga ham bevosita taalluqlidir. Chunki yuqori malakali, raqobatbardosh, zamonaviy talablarga javob bera oladigan mutaxassislarni tayyorlash uchun zamin, asosan umumiy o‘rta ta’lim maktablarida yaratiladi. Shuning uchun ham bu bo‘g‘inda matematika fanini o‘qitishga katta ahamiyat berilishi kerak. Umumiy o‘rta ta’lim maktablari matematika kursining mazmundor uslubiy yo‘nalishlaridan biri tenglamalardan iborat. Chunki tenglama va u bilan bog‘liq tushunchalar bugungi kunda umumiy o‘rta ta’lim maktablarining birinchi sinfidanoq boshlab o‘rganiladi. Bu tenglamalar ichida modul belgisi qatnashgan trigonometrik tenglamalar alohida o‘ringa ega. Chunki bunday tenglamalarni yechishda o‘quvchilar birinchidan haqiqiy sonning moduli va uning xossalaridan foydalansalar, ikkinchidan trigonometrik tenglama tushunchasi va uni yechish usullaridan foydalanadilar. Bu esa o‘quvchilarni ijodiy fikrlash qobiliyatlarini yanada rivojlantirishda muhimdir.

Haqiqiy sonlar sohasida sonning eng xarakterli tushunchalaridan biri uning moduli tushunchasidir. Bu tushunchadan nafaqat tenglamalarni yechishda, balki matematikaning turli sohalarida, jumladan, matematik analizning fundamental tushunchalaridan hisoblangan funksiyaning limitini va sonli ketmaketlikning limitini ta’riflashda foydalaniladi. Bundan tashqari undan taqribiy sonning absolyut xatosi tushunchasini ta’riflashda, mexanikaning vektor tushunchasini xarakterlashda, ratsional sonlar ustida amallar bajarish qoidalarini shakllantirishda, sonning arifmetik kvadrat ildizi va uning xossalarini o‘rganish jarayonida, ratsional ko‘rsatkichli daraja mavzusida, *n*-darajali ildizning xossalarini o‘rganishda foydalaniladi.

Haqiqiy sonning moduli uchun bir-biri bilan teng kuchli bo‘lgan turlicha ta’riflar mavjud. Quyida ulardan ba’zilarini keltiramiz.

1-ta'rif. *a* haqiqiy sonning moduli deb, $|a|$ kabi belgilanuvchi va quyidagicha aniqlangan songa aytiladi:

$$|a| = \begin{cases} a, & agar\ a > 0\ bo'lsa, \\ 0, & agar\ a = 0\ bo'lsa, \\ -a, & agar\ a < 0\ bo'lsa \end{cases}$$

2-ta'rif. *a* haqiqiy sonning moduli deb, $M(a)$ nuqtadan koordinata boshigacha bo'lgan masofaga aytiladi va u $|a|$ kabi belgilanadi.

Masalan, $|17| = 17,\ |-9| = -(-9) = 9,\ |0| = 0.$

1-misol. Agar $a = -3,\ b = 5$ bo'lsa, $|a| + |2b|$ ning qiymatini toping.

Yechish. $|a| + |2b| = |-3| + |2 \cdot 5| = -(-3) + 10 = 3 + 10 = 13.$

2-misol: Agar $a = 1,\ b = 2$ bo'lsa, $\left(-|-a|\right)^3 + 2|-b|^3 - |a| \cdot b$ ning qiymatini hisoblang.

Yechish. $\left(-|-a|\right)^3 + 2|-b|^3 - |a| \cdot b = \left(-|-1|\right)^3 + 2 \cdot |-2|^3 - |1| \cdot 2 = -1 + 16 - 2 = 13$

Haqiqiy sonning moduli bir qator xossalarga ega:

1^0. Qarama-qarshi sonlarning modullari o'zaro tengdir, ya'ni

$$|a| = |-a|$$

2^0. Chekli sondagi haqiqiy sonlar yig'indisining moduli qo'shiluvchilar modullari yig'indisidan katta emas, ya'ni

$$|a_1 + a_2 + \cdots + a_n| \le |a_1| + |a_2| + \cdots + |a_n|$$

3^0. Ikki haqiqiy son ayirmasining moduli kamayuvchi va ayriluvchi sonlar modullarining yig'indisidan katta bo'la olmaydi, ya'ni

$$|a - b| \le |a| + |b|$$

4^0. Ikki son ayirmasi yoki yig'indisining moduli shu sonlar modullari ayirmasining modulidan kichik emas va shu sonlar modullari yig'indisidan katta emas, ya'ni

$$\big||a| - |b|\big| \le |a \pm b| \le |a| + |b|$$

5^0. Chekli sondagi ko'paytuvchilar ko'paytmasining moduli ko'paytuvchilar modullarining ko'paytmasiga teng, ya'ni

$$|a_1 \cdot a_2 \cdot \ldots \cdot a_n| = |a_1| \cdot |a_2| \cdot \ldots \cdot |a_n|$$

6^0. Ikkita son bo'linmasining moduli bo'linuvchining modulini bo'luvchining moduliga bo'linmasiga teng (bo'luvchi noldan farqli bo'lganda), ya'ni

$$\left|\frac{a}{b}\right| = \frac{|a|}{|b|},\ b \neq 0.$$

Modulli ifodalarni soddalashtirishda, modulli tenglama va tengsizliklarni yechishda bu xossalardan foydalaniladi. Shuning uchun ham o'quvchilar bu xossalarni yaxshi o'zlashtirishlari kerak bo'ladi.

3-misol. $|x-3| + |x+4|$ ifodani modul belgisisiz yozing.

Yechish. Bu ifodani modul belgisisiz yozish uchun dastlab $x-3$ va $x+4$ ifodalarni har birining ishoralari o'zgarmaydigan oraliqlarini topamiz. Bunday oraliqlar $x = 3$ va $x = -4$ nuqtalar yordamida hosil qilinadi. Ular $(-\infty; -4)$, $[-4; 3)$ va $[3; +\infty)$ oraliqlardan iborat.

1) $x \in (-\infty; -4)$ uchun $x - 3 < 0$ va $x + 4 < 0$ bo'lganligidan $|x-3| = -(x-3) = 3 - x$ va $|x+4| = -(x+4) = -x - 4$ bo'lib, bu holda berilgan ifoda quyidagiga teng:

$$|x-3|+|x+4|=3-x-x-4=-2x-1;$$

2) $x\in[-4;3)$ uchun $x-3<0$ va $x+4>0$ bo'lganligidan $|x-3|=-(x-3)=3-x$ va $|x+4|=x+4$ bo'lib, bu holda berilgan ifoda quyidagiga teng bo'ladi:

$$|x-3|+|x+4|=3-x+x+4=7;$$

3) $x\in[3;+\infty)$ uchun $x-3>0$ va $x+4>0$ bo'lganligidan $|x-3|=x-3$ va $|x+4|=x+4$ bo'lib, bu holda berilgan ifoda quyidagiga teng bo'ladi:

$$|x-3|+|x+4|=x-3+x+4=2x+1.$$

Shunday qilib,

$$|x-3|+|x+4|=\begin{cases}-2x-1, & agar\ x<-4 \quad bo'lsa,\\ 7, & agar\ -4\le x<3 \quad bo'lsa,\\ 2x+1, & agar\ x\ge 3 \quad bo'lsa.\end{cases}$$

Modul qatnashgan tenglamalarni yechish uchun berilgan tenglamani modul belgisisiz yozib, uni teng kuchli tenglamalar to'plami bilan almashtiriladi. Modul qatnashgan tenglamalarni yechishda ko'pincha quyidagilardan foydalaniladi:

1) $|f(x)|=a\ \ (a\ge 0)$ ko'rinishdagi tenglama $f(x)=a$ va $f(x)=-a$ tenglamalarni yechishga keltiriladi;

2) $|f(x)|=\varphi(x)$ ko'rinishdagi tenglama $\begin{cases}f(x)=\varphi(x),\\ \varphi(x)\ge 0\end{cases}$ va $\begin{cases}f(x)=-\varphi(x),\\ \varphi(x)\ge 0\end{cases}$ sistemalarni yechishga keltiriladi;

3) $|f(x)|=|g(x)|$ ko'rinishdagi tenglama $f(x)=g(x)$ va $f(x)=-g(x)$ tenglamalarni yechishga keltiriladi.

Amalda $f(x)$, $\varphi(x)$ va $g(x)$ lar chiziqli, kvadrat, ko'rsatkichli, logarifmik va trigonometrik funksiyalar bo'lib qolishi mumkin. Trigonometrik tenglamalar mavzusini o'rganish jarayonini modul belgisi ostida trigonometrik funksiyalar qatnashgan tenglamalarni o'rganish bilan yakunlash mumkin. Bunday tenglamalarni yechishda o'quvchilardan modulning asosiy xossalarini va trigonometrik tenglamalarni yechish usullarini puxta o'zlashtirgan bo'lishlari talab qilinadi. Quyida bunday tenglamalarni yechish bilan shug'ullanamiz:

4-misol. $|\sin x+\cos x|=1$ tenglama yechilsin.

Yechish. Bu tenglama $\sin x+\cos x=1$ va $\sin x+\cos x=-1$ tenglamalarni yechishga keltiriladi. Ularni yechamiz:

1) $\sin x+\cos x=1,\ \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x\right)=1,\ \sqrt{2}\left(\cos x\cdot\cos\frac{\pi}{4}+\sin x\cdot\sin\frac{\pi}{4}\right)=1,$

$\cos\left(x-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}},\ x-\frac{\pi}{4}=\pm\frac{\pi}{4}+2\pi k,\ x=\frac{\pi}{4}\pm\frac{\pi}{4}+2\pi k,\quad k\in Z;$

2) $\sin x+\cos x=-1,\ \cos\left(x-\frac{\pi}{4}\right)=-\frac{1}{\sqrt{2}},\ x-\frac{\pi}{4}=\pm\frac{3\pi}{4}+2k\pi,\ x=\frac{\pi}{4}\pm\frac{3\pi}{4}+2k\pi,\ k\in Z.$ **5-misol**. $|\sin x|=\sin x+2\cos x$ tenglama yechilsin.

Yechish. 1) $\sin x \geq 0$ deylik. U holda berilgan tenglamadan quyidagi tenglama hosil bo'ladi: $\sin x = \sin x + 2\cos x$, $2\cos x = 0$, $\cos x = 0$, $x = \frac{\pi}{2} + 2k\pi\,(k \in Z)$. Bu $\sin x \geq 0$ shartni qanoatlantiradi.

2) $\sin x < 0$ deylik. U holda berilgan tenglamadan quyidagi tenglama hosil bo'ladi: $-\sin x = \sin x + 2\cos x$, $-2\sin x = 2\cos x$, $\sin x = -\cos x$. Bu tenglamaning yechimlaridan $\sin x < 0$ shartni faqat $x = -\frac{\pi}{4} + 2k\pi\ (k \in Z)$ qanoatlantiradi.

Javob: $x_1 = \frac{\pi}{2} + 2k\pi$, $x_2 = -\frac{\pi}{4} + 2k\pi\ (k \in Z)$.

6-misol. $|tgx| = tgx - \frac{1}{\cos x}$ tenglama yechilsin.

Yechish. Berilgan tenglamani $\frac{1}{\cos x} = tgx - |tgx|$ ko'rinishida yozamiz. Bu tenglamaning chap tomoni nolga teng bo'la olmaydi. Demak, tgx musbat ham nolga ham bo'la olmaydi., ya'ni $tgx < 0$. U holda $\frac{1}{\cos x} = tgx + tgx$, $\frac{1}{\cos x} = 2tgx$, $\frac{1}{\cos x} = 2 \cdot \frac{\sin x}{\cos x}$, $\frac{2\sin x - 1}{\cos x} = 0$,

$$\begin{cases} \sin x = \frac{1}{2}, \\ \cos x \neq 0, \\ tgx < 0 \end{cases}; \quad \begin{cases} x = \frac{\pi}{6} + 2k\pi \quad yoki \quad x = \frac{5\pi}{6} + 2k\pi \quad (k \in Z), \\ tgx < 0 \end{cases} \qquad x = \frac{5\pi}{6} + 2k\pi \quad (k \in Z).$$

Javob: $x = \frac{5\pi}{6} + 2k\pi \quad (k \in Z)$.

7-misol. $2\cos^2 x = |\sin x|$ tenglama yechilsin.

Yechish. $\cos^2 x = 1 - \sin^2 x$ bo'lgani uchun berilgan tenglamani $2 - 2\sin^2 x = |\sin x|$ ko'rinishida yozamiz. $|\sin x| = y$ deb olsak, $\sin^2 x = y^2$ bo'ladi va berilgan tenglama $2 - 2y^2 = y$ yoki $2y^2 + y - 2 = 0$ ko'rinishga keladi. Bundan $y_1 = \frac{-1 + \sqrt{17}}{4}$ ga yoki $|\sin x| = \frac{-1 + \sqrt{17}}{4}$ ga ega bo'lamiz ($y_2 = \frac{-1 - \sqrt{17}}{4}$ tenglamaning aniqlanish sohasiga kirmaydi). Bu yerda $0 < \frac{-1 + \sqrt{17}}{4} < 1$ bo'lganligi uchun oxirgi tenglamaning yechimi quyidagicha bo'ladi:

$$x = (-1)^k \arcsin \frac{-1 + \sqrt{17}}{4} + k\pi \quad (k \in Z).$$

Javob: $x = (-1)^k \arcsin \frac{-1 + \sqrt{17}}{4} + k\pi \quad (k \in Z)$.

8-misol. $3tgx = \sqrt{3}|\sin x|$ tenglama yechilsin.

Yechish. Tenglamaning har ikkala tomonining davri π ga teng. Shuning uchun uning yechimini $[0;\pi]$ oraliqda qidirish mumkin. $x_1 = 0$ tenglamani qanoatlantiradi. Demak, u yechim bo'ladi. Agar $x \neq 0$ bo'lsa, u holda $\sin x \neq 0$ bo'lib, berilgan tenglamadan $\cos x = \sqrt{3}$ ga ega

bo‘lamiz. Bu tenglama yechimga ega emas. Demak, berilgan tenglamaning barcha yechimlari $x = n\pi \ (n \in Z)$ bo‘ladi.

Javob: $x = n\pi \ (n \in Z)$.

9-misol. $2\sin^2 x = \left|\sqrt{3}tgx\right|$ tenglama yechilsin.

Yechish. Tenglamaning har ikkala qismi musbat bo‘lganligi uchun uning har ikkala tomonini kvadratga ko‘tarish bilan unga teng kuchli tenglamaga kelamiz: $4\sin^4 x = 3tg^2 x$. Agar $\sin^2 x = y$ deb olsak, u holda $4y^2 = \frac{3y}{1-y}$, $4y^2 - 4y^3 = 3y$, $y(4y^2 - 4y + 3) = 0$; $y = 0$ ($4y^2 - 4y + 3$ tenglama yechimga ega emas). Demak, $\sin^2 x = 0$ bo‘lib, bundan $x = k\pi \ (k \in Z)$.

Javob: $x = k\pi \ (k \in Z)$.

10-misol. $\left|\cos x\right| = \cos x - 2\sin x$ tenglama yechilsin.

Yechish. $\cos x \geq 0$ bo‘lsa, $\cos x = \cos x - 2\sin x$, $\sin x = 0$, $x = 2k\pi$, $k \in Z$; $\cos x < 0$ bo‘lsa, $-\cos x = \cos x - 2\sin x$, $\cos x = \sin x$, $tgx = 1$, $x = \frac{5\pi}{4} + 2k\pi$, $k \in Z$.

Javob: $x_1 = 2k\pi$, $x_2 = \frac{5\pi}{4} + 2k\pi \ (k \in Z)$.

11-misol. $\cos x = \left|\cos x\right|\left(x + 1,5\right)^2$ tenglama yechilsin.

Yechish. 1) $\cos x = 0$ bo‘lsa, 0=0 bo‘lib, bu holda $x = \frac{\pi}{2} + k\pi \ (k \in Z)$.

2) $\cos x > 0$ bo‘lsa, $\cos x = \cos x(x + 1,5)^2$, $(x + 1,5)^2 = 1$, $x + 1,5 = \pm 1$; $x_1 = -0,5$, $x_2 = -2,5$. $\cos(-0,5) = \cos 0,5 > 0$, chunki $0 < 0,5 < \frac{\pi}{2}$; $\cos(-2,5) = \cos 2,5 < 0$, chunki $\frac{\pi}{2} < 2,5 < \pi$. Demak, $x = 2,5$ ildiz bo‘la olmaydi.

3) $\cos x < 0$ bo‘lsa, $-1 = (x + 1,5)^2$ bo‘ladi va bu yechimga ega emas.

Javob: $x_1 = -0,5$; $x_2 = \frac{\pi}{2} + k\pi \ (k \in Z)$.

12-misol. $\cos x = \left|\sin x\right|$ tenglama yechilsin.

Yechish. 1) $\sin x \geq 0$ bo‘lsa, $\cos x = \sin x$, $tgx = 1$, $x = \frac{\pi}{4} + 2k\pi \ (k \in Z)$.

2) $\sin x < 0$ bo‘lsa, $\cos x = -\sin x$, $tgx = -1$, $x = -\frac{\pi}{4} + 2k\pi \ (k \in Z)$.

Javob: $\frac{\pi}{4} + 2k\pi$, $-\frac{\pi}{4} + 2k\pi \ (k \in Z)$.

13-misol. $\sqrt{3}\sin x = \left|\cos x\right|$ tenglama yechilsin.

Yechish. $\sqrt{3}\sin x = \left|\cos x\right| \Leftrightarrow \begin{cases} 3\sin^2 x = \cos^2 x, \\ \sin x \geq 0. \end{cases} \Leftrightarrow \begin{cases} 4\sin^2 x = 1, \\ \sin x \geq 0 \end{cases} \Leftrightarrow \begin{cases} \sin x = \frac{1}{2}, \\ \sin x \geq 0 \end{cases} \Leftrightarrow$

$\Leftrightarrow x = (-1)^k \frac{\pi}{6} + k\pi \ (k \in Z)$.

Javob: $x=(-1)^k\frac{\pi}{6}+k\pi\ (k\in Z)$.

14-misol. $\cos x=tgx|\cos x|$ tenglama yechilsin.

Yechish. Tenglamaning har ikkala tomonini kvadratga ko'taramiz. $\cos^2 x=tg^2x\cdot\cos^2 x$. Tenglamaning aniqlanish sohasi $\cos x\neq 0$ bo'lgani uchun $\cos^2 x>0$ bo'ladi. Shuning uchun oxirgi tenglamadan $tg^2x=1$ ni topamiz. Bundan esa $tgx=\pm 1,\ x_1=\frac{\pi}{4}+k\pi,\ x=\frac{\pi}{4}+k\pi\ (k\in Z)$.

Kvadratga ko'tarish jarayonida chet ildizlar paydo bo'lib qolishi mumkin. Shuning uchun $\cos x$ va tgx lar bir xil ishoraga ega bo'ladigan ildizlarni ajratishimiz kerak. Ular: $x=\frac{\pi}{4}+2k\pi$ va $x=\frac{3\pi}{4}+2k\pi\ (k\in Z)$.

Javob: $x=\frac{\pi}{4}+2k\pi$, $x=\frac{3\pi}{4}+2k\pi\ (k\in Z)$.

15-misol. $|\sin x|+\sin x(x-4)^2=0$ tenglama yechilsin.

Yechish. $\sin x>0$ bo'lsa, tenglamaning chap tomoni musbat va bu holda tenglama yechimga ega emas. $\sin x=0$ bo'lsa, tenglamaning ildizi $x=k\pi\ (k\in Z)$ bo'ladi. $\sin x<0$ bo'lsa, u holda quyidagiga ega bo'lamiz: $-\sin x+\sin x(x-4)^2=0,\ \sin x\cdot\left[(x-4)^2-1\right]=0,\ (x-4)^2-1=0,\ (x-4)^2=1,$ $x-4=\pm 1,\ x_1=5,\ x_2=3$ bo'ladi. $\pi<5<2\pi$ bo'lgani uchun $\sin 5<0$ va $\frac{\pi}{2}<3<\pi$ bo'lgani uchun $\sin 3>0$. Shuning uchun x=5 berilgan tenglamaning ildizi bo'ladi.

Javob: $x=5,\ x=k\pi\ (k\in Z)$.

16-misoi. $4^{|x-2|\sin x}=2^{x|\sin x|}$ tenglama yechilsin.

Yechish. $4^{|x-2|\sin x}=2^{x|\sin x|},\ 2^{2|x-2|\sin x}=2^{x|\sin x|},\ 2|x-2|\sin x=x|\sin x|$.

1) Agar $\sin x=0$ bo'lsa, u holda $x=k\pi\ (k\in Z)$.

2) Agar $\sin x>0$ bo'lsa, u holda $2|x-2|\sin x=x\sin x,\ 2|x-2|=x;$

$\begin{cases}3x^2-16x+16=0,\\ x\geq 0\end{cases}$; $x=4$ yoki $x=\frac{4}{3}$.

$\frac{3\pi}{2}<4<2\pi$ bo'lgani uchun $\sin 4<0$. Demak, $x=4$ berilgan tenglamaning ildizi bo'la olmaydi. $0<\frac{4}{3}<\frac{\pi}{2}$ bo'lganligi uchun $\sin\frac{4}{3}>0$. Demak, $x=\frac{4}{3}$ berilgan tenglamaning ildizi ekan.

Javob. $x=k\pi\ (k\in Z),\ x=\frac{4}{3}$.

17-misol. $\sin x=tgx|\sin x|$ tenglama yechilsin.

Yechish. 1) $\sin x=0$ bo'lsa, $x=k\pi\ (k\in Z)$.

2) $\sin x>0$ bo'lsa, $\sin x=tgx\cdot\sin x,\ tgx=1,\ x=\frac{\pi}{4}+k\pi\ (k\in Z)$. Bu yechimlardan $\sin x>0$ shartni $x=\frac{\pi}{4}+2k\pi\ (k\in Z)$ qanoatlantiradi.

2) $\sin x < 0$ bo‘lsa, $\sin x = -tgx \cdot \sin x$, $tgx = -1$, $x_{1,2} = \pm\frac{\pi}{4} + 2k\pi$.

Javob: $x_{1,2} = \pm\frac{\pi}{4} + 2k\pi$, $x_3 = k\pi$ $(k \in Z)$.

18-misol. $\frac{\cos x}{\left(x+\frac{3}{2}\right)^2} = |\cos x|$ tenglama yechilsin.

Yechish. Tenglamaning chap tomonining maxraji doimo musbat bo‘ladi. Shuning uchun berilgan tenglamaning yechimi $\cos x \geq 0$ shartni qanoatlantirishi kerak. Ammo $|\cos x| = \cos x$ ekanligidan tenglamani

$$\frac{\cos x}{\left(x+\frac{3}{2}\right)^2} = \cos x$$

ko‘rinishida yozish mumkin. Bu tenglamaning aniqlanish sohasi $x \neq -\frac{3}{2}$ sonlardan iborat. Shuning uchun oxirgi tenglama o‘zining aniqlanish sohasida

$$\cos x = 0 \text{ va } \frac{1}{\left(x+\frac{3}{2}\right)^2} = 1$$

tenglamalarga teng kuchlidir. Bu tenglamalarni yechamiz:

1) $\cos x = 0,\ x = \frac{\pi}{2} + k\pi,\ k \in Z.$

2) $\frac{1}{\left(x+\frac{3}{2}\right)^2} = 1,\ \left(x+\frac{3}{2}\right)^2 = 1,\ x+\frac{3}{2} = -1,\ x+\frac{3}{2} = 1;$ yoki $x = -\frac{5}{2}$ va $x = -\frac{1}{2}$. x ning topil-

gan bu qiymatlaridan qaysinisi $\cos x \geq 0$ va $x \neq -\frac{3}{2}$ shartni qanoatlantirishini aniqlaymiz.

Ko‘rish osonki, $x = \frac{\pi}{2} + k\pi,\ k \in Z$ yechimlar yuqoridagi shartni qanoatlantiradi, chunki $\frac{\pi}{2} + k\pi$ yechimlar k ning hech bir qiymatida $-\frac{3}{2}$ ga teng emas. $0 < \frac{1}{2} < \frac{\pi}{2}$ bo‘lganligi uchun $\cos\left(-\frac{1}{2}\right) = \cos\frac{1}{2} > 0$ bo‘ladi. Demak, $x = -\frac{1}{2}$ son $\cos x \geq 0$ va $x \neq -\frac{3}{2}$ shartlarni qanoatlantiradi. Bu esa $x = -\frac{1}{2}$ berilgan tenglamaning yechimi ekanligini bildiradi.

Endi $x = -\frac{5}{2}$ ni tekshiramiz. $-\pi < -\frac{5}{2} < -\frac{\pi}{2}$ bo‘lganligi uchun, $\cos\left(-\frac{5}{2}\right) < 0$ va $x = -\frac{5}{2}$ berilgan tenglamaning ildizi bo‘la olmaydi. Demak, berilgan tenglamaning yechimlari $x = -\frac{1}{2}$ va $x = \frac{\pi}{2} + k\pi,\ k \in Z$ lardan iborat.

Javob: $x=-\frac{1}{2}$; $x=\frac{\pi}{2}+k\pi,\ k\in Z$.

Bu yerda ko'rib o'tilgan va ularga o'xshash tenglamalarni yechish jarayonida o'quvchilarning trigonometrik tenglamalar bo'yicha bilimlari mustahkamlanadi. Bundan tashqari o'quvchilarda haqiqiy sonning moduli va uning xossalarini tenglamalarni yechishda qo'llash malakalari shakllanadi hamda pirovard natijada ta'limning samarali bo'lishiga erishiladi.

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SOME CONSIDERATIONS FOR SOLVING TRIGONOMETRIC EQUATIONS WITH MODULUS SIGN

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It is known that, in the recent years, the Republic has been paying great attention to the radical reform of the education system, including mathematics education. A clear example of this is the Presidential Decree on measures to further develop mathematics education and science in the country. The tasks set in this resolution are directly relevant to the general secondary education sector, which is one of the most important links in the system of continuing education. Because the basis to train highly qualified, competitive, modern specialists will be created, mainly in general secondary schools. Therefore, it is important to pay special attention to teaching of mathematics in this area. As you know, one of the most important methodological areas of general secondary school mathematics is equations. Because the equation and the concepts associated with it are studied today from the first grade of general secondary school. The trigonometric equations with the modulus sign have a special place in these equations. Because in solving such

equations, students first use the modulus of a real number and its properties, and secondly, the concept of a trigonometric equation and methods of solving it. This is important to further develop students' creative thinking skills.

One of the most characteristic concepts of a number in the set of real numbers is its concept of modulus. This concept is used not only in solving equations, but also in various fields of mathematics, including the definition of the limit of a function and the limit of a numerical sequence calculated from the fundamental concepts of mathematical analysis. In addition, the concept of modulus is used in defining the concept of absolute error of an approximate number, in characterizing the vector concept of mechanics, in the formation of rules for operations on rational numbers, in the study of the arithmetic square root of a number and its properties, on the topic of rational exponent.

There are different definitions for the absolute value of a real number that are equally strong. Here are some of them:

Definition 1. The absolute value or modulus of a real number a is the non-negative value defined as $|a|$:

$$|a| = \begin{cases} a, & agar\ a > 0\ bo'lsa, \\ 0, & agar\ a = 0\ bo'lsa, \\ -a, & agar\ a < 0\ bo'lsa \end{cases}$$

Definition 2. The modulus of a real number a, the distance from the point $M(a)$ to the origin, and is denoted by it $|a|$

The modulus of a real number has a number of properties. They are presented in the article without proof. These properties are used to simplify modular expressions, to solve modular equations and inequalities. That's why students need to master these qualities.

To solve the equations in which the module is involved, write the given equation without the module symbol, replace it with a set of equations of equal strength, and solve it.

The properties of the module are also used in solving trigonometric equations involving the module symbol. The article describes the experiments on the study of equations involving the module symbol.

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