UO‘K 371.3

UMUMIY O‘RTA TA’LIM MAKTABLARI VA AKADEMIK LITSEYLARDA TENGLAMALAR SISTEMASINI O‘RGANISH USLUBLARI

A.Axlimirzayev

Maqolada umumiy o‘rta ta’lim maktablari va akademik litseylar matematika kursida o‘rganiladigan ikki noma’lumli ikkita tenglamalar sistemasi, ularning turlari va yechish usullari misollar yordamida bayon qilingan. Unda dastlab ikki noma’lumli ikkita chiziqli tenglamalar sistemasini yechishning o‘rniga qo‘yish usuli, qo‘shish usuli, taqqoslash usuli, Kramer usuli va grafik usullarning mazmun-mohiyati yoritib berilgan.

Kalit so‘zlar: *tenglama, tenglamalar sistemasi, tenglamalar sistemasini yechish, o‘rniga qo‘yish usuli, qo‘shish usuli, Kramer usuli, grafik usul, sun’iy usul.*

В статье посредством примеров описана система двух уравнений с двумя неизвестными, виды и методы их решения которых изучаются в курсе математики средних общеобразовательных школ и академических лицеев. В ней освещено содержание метода подстановки, метода сложения, метода сравнения, метода Крамера и графического метода, которые применяются при решении систем линейных уравнения с двумя неизвестными.

Ключевые слова: *уравнение, система уравнений, решение системы уравнений, метод подстановки, метод сложения, метод Крамера, графический метод, искусственный метод.*

Ma’lumki, matematikaning bir qator tushunchalari amaliyotning u yoki bu masalasini yechish zaruratidan kelib chiqqan. Tenglama yoki tenglamalar sistemasi tushunchalari ham ana shunday amaliy masalalarning matematik modelidan iborat. Ayniqsa, uch noma’lumli uchta chiziqli tenglamalar sistemasi ko‘plab iqtisodiy masalalarning matematik modelidan iborat. Shuning uchun ham o‘quvchilar eng avvalo tenglama va tenglamalar sistemasini yechish malakalarini puxta egallashlari kerak bo‘ladi.

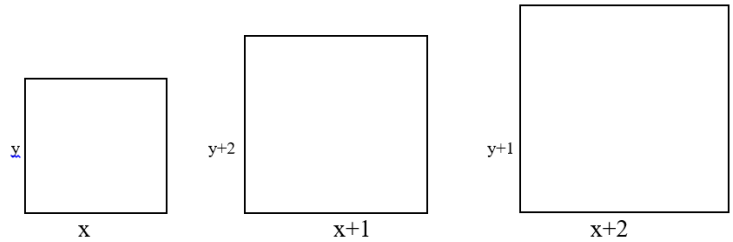
Tenglamalar sistemasini o‘rganishda tenglama, uning yechimi, tenglamalarning teng kuchliligi tushunchalari muhim ahamiyatga ega. Tenglamalar sistemasini o‘rganishni unga olib keluvchi quyidagi masala bilan boshlash mumkin:

Masala: To‘g‘ri to‘rtburchakning uzunligi 1 m ga, eni 2 m ga uzaytirilganda, uning yuzi 77 kv.m ga ortishi, eni 1 m ga, uzunligi 2 m ga uzaytirilganida, uning yuzi 68 kv.m ga ortishi uchun uning o‘lchamlari qanday bo‘lishi kerak?

Yechish: Bu masalada ikkita kattalikni, ya’ni to‘g‘ri to‘rtburchak enini va bo‘yini topish talab qilinyapti. Masalani yechish uchun uning uzunligini x bilan, enini y bilan belgilaymiz. U holda to‘g‘ri to‘rtburchakning dastlabki yuzi xy bo‘ladi. To‘g‘ri to‘rtburchakning tomonlari o‘zgarganda esa yuzalar mos ravishda quyidagicha bo‘ladi:

$$(x+1)(y+2) \text{ va } (x+2)(y+1)$$

Har uchala to‘g‘ri to‘rtburchakni yasaymiz.



1-rasm 2-rasm 3-rasm

Masalaning shartiga asosan quyidagiga ega bo‘lamiz:

$$(x\ +1)(y+2) = xy\ +77$$
$$(x\ +2)(y+1) = xy\ +68$$

Mazkur tenglamalarning har biri birinchi darajali ikki noma’lumli tenglamadir. Bu tenglamalarda x va y lar biror sonni ifodalaydi. Bunday holda yuqoridagi tenglamalar sistema hosil qiladi deyiladi va u quyidagicha yoziladi:

$$\begin{cases}(x\ +1)(\ y\ +2) = xy+77\\(x\ +2)(\ y\ +1) = xy+68\end{cases} \text{ yoki } \begin{cases}2x+y=75\\x+2y=66\end{cases} \quad (A)$$

Agar (A) sistemadagi x o‘rniga 30 ni, y o‘rniga 10 ni qo‘ysak, sistemaning hech bir tenglamasi to‘g‘ri tenglikka aylanmaydi. Agar *x*=30 va *y*=15 deb olsak, bu holda ular sistemaning birinchi tenglamasini qanoatlantiradi, ammo ikkinchi tenglamasini qanoatlantirmaydi.

Sistemaning har ikkala tenglamasini bir vaqtda qanoatlantiradigan x va *y* ning qiymatlarini topish uchun quyidagicha ish tutamiz.

x ni ma’lum deb olib, sistemaning birinchi tenglamasidan $y = 75 - 2x$ ni topib, uni sistemaning ikkinchi tenglamasiga qo‘yamiz va u quyidagi bir noma’lumli tenglamaga keladi:

$$x + 2(75 - 2x) = 66$$

Bundan $x + 150 - 4x = 66$, $-3x = -84$, $x = 28$ kelib chiqadi. x ning bu qiymatini $y = 75 - 2x$ ga qo‘yib, $y = 19$ ni topamiz. Demak, sistemaning yechimi $x = 28$, $y = 19$ dan iborat ekan. Shunday qilib, yuqoridagi masalani yechish jarayonida biz birinchi darajali ikki no‘malumli ikkita tenglamalar sistemasiga duch keldik. Bunday sistemalarni ikki no‘malumli ikkita chiziqli tenglamalar sistemasi deyiladi va u umumiy holda quyidagicha yoziladi:

$$\begin{cases}a_1x + b_1y = c_1\\a_2x + b_2y = c_2\end{cases} (1)$$

Yuqoridagi masalani yechishda qo‘llanilgan usul o‘rniga qo‘yish usuli deyiladi.

(1) ko‘rinishdagi sistemani qo‘shish usuli bilan ham yechish mumkin. Bu quyidagicha bajariladi.

Berilgan sistemaning birinchi tenglamasini b_2 ga, ikkinchi tenglamasini $-b_1$ ga hadma-had ko‘paytiramiz va ularni hadma-had qo‘shamiz:

$$\begin{cases}a_1x + b_1y = c_1, \mid b_2\\a_2x + b_2y = c_2. \mid -b_1\end{cases}; \begin{cases}a_1b_2x + b_1b_2y = b_2c_1,\\-a_2b_1x + b_1b_2y = -b_1c_2.\end{cases};$$
$$a_1b_2x - a_2b_1x = b_2c_1 - b_1c_2\ ,\ (a_1b_2 - a_2b_1)x = b_2c_1 - b_1c_2,$$

$$x = \frac{b_2c_1 - b_1c_2}{a_1b_2 - a_2b_1} \text{ (bu yerda } a_1b_2 - a_2b_1 \neq 0).$$

y ni topish uchun sistemaning birinchi tenglamasini $-a_2$ ga, ikkinchi tenglamasini a_1 ga ko‘paytirib, ularni hadma-had qo‘shamiz:

$$\begin{cases}a_1x + b_1y = c_1 \mid -a_2\\a_2x + b_2y = c_2 \mid a_1\end{cases}; \begin{cases}-a_1a_2x - a_2b_1y = -a_2c_1,\\a_1a_2x + a_1b_2y = a_1c_2\end{cases};$$
$$a_1b_2y - a_2b_1y = a_1c_2 - a_2c_1,\ (a_1b_2 - a_2b_1)y = a_1c_2 - a_2c_1,$$

$$y = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}\text{, (bu yerda } a_1b_2 - a_2b_1 \neq 0).$$

Bu usulni qo'llashga doir quyidagi misolni ko'ramiz:

1-misol: $\begin{cases}2x+3y=49,\\3x+2y=46.\end{cases}$ sistema yechilsin.

Yechish: Bu sistemani yechish uchun dastlab uning birinchi tenglamasini -2 ga, ikkinchi tenglamasini 3 ga, so'ngra birinchi tenglamasini 3 ga, ikkinchi tenglamasini -2 ga ko'paytirib, ularni hadma-had qo'shamiz.

$$\begin{cases}2x+3y=49,|-2\,|3\\3x+2y=46.\,|\,3\,|-2\end{cases};\ \begin{cases}5x=40,\\5y=55.\end{cases};\ \begin{cases}x=8,\\y=11.\end{cases}$$

(1) ko'rinishdagi sistemani taqqoslash usuli bilan ham yechish mumkin. Bu quyidagicha amalga oshiriladi. Sistemaning har bir tenglamasidan y ning ifodasini x orqali yoki x ning ifodasini y orqali topamiz va ularni tenglaymiz. Bunga quyidagi misolni keltiramiz:

2-misol: $\begin{cases}2x+3y=7,\\3x-2y=4.\end{cases}$ sistema yechilsin.

Yechish: $2x+3y=7, 3y=7-2x, y=\frac{7-2x}{3}$; $3x-2y=4$, $2y=3x-4$, $y=\frac{3x-4}{2}$.

Demak, biz quyidagi tenglamaga ega bo'lamiz:

$$\frac{7-2x}{3}=\frac{3x-4}{2}, 2(7-2x)=3(3x-4), 14-4x=9x-12, 13x=26,$$

$x=2$.

x ning bu qiymatini o'rniga qo'yamiz:

$$y=\frac{7-2x}{3}=\frac{7-2\cdot 2}{3}=\frac{3}{3}=1.$$

Demak, $x=2, y=1$.

(1) tenglamalar sistemasini Kramer formulalari deb ataluvchi formulalardan foydalanib ham yechish mumkin. Bu formulalar quyidagicha:

$$x=\frac{D_x}{D},\ y=\frac{D_y}{D}.$$

Bu yerda $D=\begin{vmatrix}a_1 & b_1\\a_2 & b_2\end{vmatrix}=a_1b_2-a_2b_1$;

$$D_x=\begin{vmatrix}c_1 & b_1\\c_2 & b_2\end{vmatrix}=c_1b_2-c_2b_1;\ D_y=\begin{vmatrix}a_1 & c_1\\a_2 & c_2\end{vmatrix}=a_1c_2-a_2c_1$$

bo'lib, ular determinantlar deb ataladi. Kramer formulalarini qo'llashga misol ko'ramiz.

3-misol: $\begin{cases}3x+4y=10,\\4x+3y=11.\end{cases}$ sistema yechilsin.

Yechish: D=$\begin{vmatrix}3 & 4\\4 & 3\end{vmatrix}=9-16=-7$; $D_x=\begin{vmatrix}10 & 4\\11 & 3\end{vmatrix}=30-44=-14$,

$D_y=\begin{vmatrix}3 & 10\\4 & 11\end{vmatrix}=33-40=-7$; $x=\frac{D_x}{D}=\frac{-14}{-7}=2$, $y=\frac{D_y}{D}=\frac{-7}{-7}=1$

Demak, $x=2, y=1$.

(1) tenglamalar sistemasini matritsalar usuli bilan ham yechish mumkin. Buning uchun matritsa tushunchasi, matritsalar ustida amallar, teskari matritsa tushunchasi hamda tenglamalar sistemasining matritsaviy yozuvi bilan o'quvchilar tanishgan bo'lishlari kerak. Bu tushunchalar oliy o'quv yutuqlarida oliy matematika kursida o'rganiladi. Shuning uchun ham bu yerda biz matritsalar usuliga to'xtalmaymiz.

(1) tenglamalar sistemasini grafik usulda ham yechish mumkin. Bunda sistemaning har bir tenglamasidagi y ni x orqali ifodalaymiz va ularni funksiya sifatida qarab, bitta chizmada grafiklarini yasaymiz. Bu grafiklarning kesishgan nuqtasi koordinatalari sistemaning yechimidan iborat bo‘ladi. Quyida bunga misol keltiramiz.

4-misol: $\begin{cases}2x+y=6\\3x-y=4\end{cases}$ sistema yechilsin.

Yechish: Sistemaning birinchi tenglamasidan y =6-2x, ikkinchi tenglamasidan esa $y=3x-4$ ni yozamiz. Bularning grafiklarini bitta chizmada yasaymiz (1-chizma).

Grafiklar A(2; 2) nuqtada kesishishini ko‘rish mumkin.

Agar ikki noma’lumli ikkita tenglamalar sistemasining bitta tenglamasi birinchi darajali, ikkinchi tenglamasi yuqori darajali bo‘lsa, u holda yuqoridagi usullardan, asosan, o‘rniga qo‘yish usulidangina foydalanish mumkin. Bunga quyidagi misolni keltiramiz:

5-misol: $\begin{cases}y-x=1,\\2x^2-xy-y^2+2x-2y+6=0\end{cases}$ sistema yechilsin.

Yechish: Sistemaning birinchi tenglamasidan $y=x+1$ topamiz va buni sistemaning ikkinchi tenglamasiga qo‘yamiz:

$$2x^2-x(x+1)-(x+1)^2+2x-2(x+1)+6=0\ ,$$
$$2x^2-x^2-x-x^2-2x-1+2x-2x-2+6=0,$$

$-3x+3=0, x=1. x=1$ bo‘lsa, u holda $y=x+1=1+1=2$ bo‘ladi.

Demak, $x=1, y=2$.

Ba’zi hollarda sistemaning har ikkala tenglamasi ikkinchi darajali bo‘lishi mumkin. Bunday sistemalar ham asosan o‘rniga qo‘yish usuli bilan yechiladi. Ammo ularda dastlab o‘rniga qo‘yish usulini qo‘llash mumkin bo‘lgan holga keltiriladi. Sistemani yechishning bu usulini sun’iy usul deb ham ataladi. Bunga quyidagi misolni keltirish mumkin:

6-misol: $\begin{cases}x^2-xy-y^2=5\\2x^2+xy-10y^2=11\end{cases}$ sistema yechilsin.

Yechish: Sistemaning birinchi tenglamasini 11 ga, ikkinchi tenglamasini -5 ga hadma-had ko‘paytiramiz va hosil bo‘lgan tenglamalarni hadma-had qo‘shamiz.

$$\begin{cases}x^2-xy-y^2=5, |11\\2x^2+xy-10y^2=11\ |-5\end{cases}; \begin{cases}11x^2-11xy-11y^2=55,\\-10x^2-5xy+50y^2=-55\end{cases};$$

$x^2-16xy+39y^2=0$. Bu tenglama ikkilnchi darajali bir jinsli tenglamadir. Uni yechish uchun har ikkala tomonini hadma-had y^2 $(y\neq 0)$ ga bo‘lamiz. $y\neq 0$ bo‘lishi ravshan (Buni isbot qilish qiyin emas).

$$\frac{x^2}{y^2}-\frac{16xy}{y^2}+\frac{39y^2}{y^2}=0;\ \left(\frac{x}{y}\right)^2-\frac{16x}{y}+39=0$$

Bu tenglama $\frac{x}{y}$ ga nisbatan kvadrat tenglamadir. Uni yechib $\frac{x}{y}=13, \frac{x}{y}=3$ larni topamiz. Bulardan x_1=13y, $x_2=3y$ larni yozish mumkin. x ning y orqali ifodalarini sistemaning istalgan bir tenglamasiga, masalan, $x^2-xy-y^2=5$ tenglamaga qo‘yamiz.

$$1)\ x_1=13y,\ (13y)^2-13y\cdot y-y^2=5\ ,\ 155y^2=5,\ y^2=\frac{5}{155}=\frac{1}{31}.$$

Bundan $y_1=\frac{1}{\sqrt{31}}$ va $y_2=-\frac{1}{\sqrt{31}}$ Demak, x_1=$\frac{13}{\sqrt{31}}$, , $x_2=-\frac{13}{\sqrt{31}}$;

$$2)\ x_2 = 3y, (3y)^2 - 3y \cdot y - y^2 = 5, 5y^2 = 5;\ y^2 = 1, y_3 = 1, y_4 = -1.$$

Demak, $x_3 = 3, x_4 = -3$. Shunday qilib, berilgan sistemalardan iborat to'rtta yechimga ega ekan. $(\frac{13}{\sqrt{31}}, \frac{1}{\sqrt{31}}), \left(-\frac{13}{\sqrt{31}}, -\frac{13}{\sqrt{31}}\right)$, (3; 1), (-3; -1).

Amalda $\begin{cases} x + y = a, \\ xy = b \end{cases}$ ko'rinishdagi sistemalar ham uchrab turadi. Bunday sistemalarni o'rniga qo'yish usuli bilan ham yechish mumkin. Bunda birinchi tenglamadan y ni x orqali yoki x ni y orqali ifodalab, ikkinchi tenglamadagi x ni yoki y ni o'rniga qo'yiladi. Ya'ni =a- x, x(a- x)=b, ax -x^2=b yoki $x^2 - ax + b = 0$. Hosil bo'lgan kvadrat tenglama yechilib, x ning qiymatlari topiladi, so'ngra x ning topilgan qiymatlarini $y = a - x$ ga qo'yib y ning qiymatlari topiladi. Lekin bunday sistemalarni x va y larni keltirilgan kvadrat tenglamaning ildizlari deb qarab yechish qulaydir.

Bunga quyidagi misolni keltiramiz:

7-misol: $\begin{cases} x + y = 30, \\ xy = 209 \end{cases}$ sistema yechilsin.

Yechish: Agar biz x va y larni ozod hadi 209 ga, ikkinchi hadining koeffitsiyentini -30 ga teng bo'lgan $z^2 - 30z + 209 = 0$ kvadrat tenglamaning ildizlari deb qarab, uni yechsak, x =19, y =11 yoki x=11, y =19 larni topamiz. Bular berilgan sistemaning yechimlaridan iborat bo'ladi.

Ba'zi bir tenglamalar sistemasi, masalan, $\begin{cases} x^2 + y^2 = a \\ xy = b \end{cases}$ ko'rinishdagi sistemalar ma'lum bir ayniy al-mashtirishlardan so'ng $\begin{cases} x + y = a \\ xy = b \end{cases}$ ko'rinishdagi sistemaga keltiriladi. Bunga quyidagi misolni keltirish mumkin:

8-misol: $\begin{cases} x^2 + y^2 = 10, \\ xy = 3 \end{cases}$ sistema yechilsin.

Yechish: Sistemaning ikkinchi tenglamasining har ikkala tomonini 2 ga ko'paytiramiz va birinchi tenglamaga hadma-had qo'shamiz.

$x^2 + 2xy + y^2 = 16$ yoki $(x + y)^2 = 16$ tenglamaga ega bo'lamiz.

Bundan $x + y = 4$ va $x + y = -4$ larni topamiz. Natijada biz quyidagi ikkita sistemaga ega bo'lamiz:

$$1) \begin{cases} x + y = 4, \\ xy = 3. \end{cases}; 2) \begin{cases} x + y = -4, \\ xy = 3. \end{cases}$$

Bularning birinchisidan $x_1 = 1, y_1 = 3$ va $x_2 = 3, y_2 = 1$ larni, ikkinchisidan esa $x_3 = -1, y_3 = -3$ va $x_4 = -3, y_4 = -1$ larni topamiz. Demak, berilgan sistema (1:3), (3:1), (-1:-3), (-3:-1) lardan iborat 4 ta yechimga ega ekan.

Ba'zan $\begin{cases} x^4 + y^4 = a_1 \\ x + y = b_1 \end{cases}$ ko'rinishdagi sistemalar ham uchraydi. Bunday sistemalar ham ma'lum ayniy al-mashtirishlardan so'ng $\begin{cases} x + y = a, \\ xy = b \end{cases}$ ko'rinishdagi sistemalarga keltiriladi. Bunga quyidagi sistemani misol qilib keltirish mumkin.

9-misol: $\begin{cases} x^4 + y^4 = 17, \\ x + y = 3 \end{cases}$ sistema yechilsin.

Yechish: Bu sistemani yechish uchun uni oldingi sistema ko'rinishiga keltirishga harakat qilamiz. Buning uchun $x^2 + y^2 = (x + y)^2 - 2xy$ ayniyatdan foydalanamiz. Bunda $x + y = 3$ dan foydalanamiz. Unga asosan, $x^2 + y^2 = 9 - 2xy$. Huddi shunday $x^4 + y^4 = (x^2 + y^2)^2 - 2x^2y^2$.

Endi $x^2 + y^2 = 9 - 2xy$ ekanligini e'tiborga olsak, u holda

$$x^4 + y^4 = (9 - 2xy)^2 - 2x^2y^2 = 81 - 36xy + 4x^2y^2 - 2x^2y^2 \text{ bo'ladi.}$$

Shunday qilib, dastlabki sistema quyidagi ko‘rinishga keladi:

$$\begin{cases} 2x^2y.^2 - 36xy + 81 = 17, \\ x + y = 3 \end{cases} \text{ yoki } \begin{cases} x^2y^2 - 18xy + 32 = 0, \\ x + y = 3. \end{cases}$$

Agar biz $xy = z$ deb olsak, u holda sistemaning birinchi tenglamasi $z^2 - 18z + 32 = 0$ ko‘rinishga keladi. Bu kvadarat tenglama bo‘lib, uning ildizlari $z_1 = 2$ va $z_2 = 16$ bo‘ladi. Shunday qilib, biz quyidagi sistemalarga kelamiz:

$$1) \begin{cases} x + y = 3, \\ xy = 16 \end{cases}; 2) \begin{cases} x + y = 3, \\ xy = 2 \end{cases}$$

Bu sistemalardan birinchisi yechimga ega emas. Ikkinchi sistemadan esa $x_1 = 1,\ y_1 = 2$ va $x_2 = 2, y_2 = 1$ larni topamiz. Demak, berilgan sistema (1; 2) va (2; 1) lardan iborat ikkita yechimga ega ekan.

$\begin{cases} x^5 + y^5 = a_1, \\ x + y = b_1. \end{cases}$ ko‘rinishdagi sistema ham ma’lum bir almashtirishlardan so‘ng $\begin{cases} x + y = a, \\ xy = b. \end{cases}$ ko‘rinishdagi sistemaga keltiriladi. Bunga quyidagi misolni keltirish mumkin:

10-misol: $\begin{cases} x^5 + y^5 = 275, \\ x + y = 5 \end{cases}$ sistema yechilsin.

Yechish: Sistemadagi ikkinchi tenglamaning har ikkala tomonini beshinchi darajaga ko‘taramiz.

$(x + y)^5 = 5^5,\ x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5 = 3125.$

Hosil bo‘lgan tenglamadan sistemaning birinchi tenglamasini hadma-had ayiramiz.

$$5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 = 2850, x^4y + 2x^3y^2 + 2x^2y^3 + xy^4 = 570$$

tenglama hosil bo‘ladi. Bu tenglamaning chap tomonini ko‘paytuvchilarga ajratamiz.

$$xy(x^3 + 2x^2y + 2xy^2 + y^3) = 570, xy[(x^3 + y^3) + 2xy(x + y)] = 570,$$
$$xy(x + y)(x^2 - xy + y^2 + 2xy) = 570,\ xy \cdot 5[(x + y)^2 - xy] = 570,$$
$$xy[(x + y)^2 - xy] = 114,\ xy(25 - xy) = 114,\ 25xy - (xy)^2 = 114,$$
$$(xy)^2 - 25xy + 114 = 0.$$

Bu xy ga nisbatan kvadrat tenglamadir. Uni yechib, $xy = 19, xy = 6$ larni topamiz. Demak, biz quyidagi ikkita sistemaga ega bo‘ldik:

$$1) \begin{cases} x + y = 5, \\ xy = 19. \end{cases}; 2) \begin{cases} x + y = 5, \\ xy = 6. \end{cases}$$

Bu sistemalardan birinchisi yechimga ega emas. Ikkinchi sistemadan $x_1 = 3,\ y_1 = 2$ va $x_2 = 3, y_2 = 2$ larni topamiz. Demak, berilgan sistema (3; 2) va (2; 3) lardan iborat ikkita yechimga ega ekan.

11-misol: $\begin{cases} x^3 + y^3 = 8 \\ x^2 + y^2 = 4 \end{cases}$ sistema yechilsin.

Yechish: $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$ va $x^2 + y^2 = (x + y)^2 - 2xy$ ayniyatlardan foydalanamiz. U holda $x^3 + y^3 = (x + y)^3 - 3xy(x + y)$ va $x^2 + y^2 = (x + y)^2 - 2xy$ bo‘lganligi uchun ulardan $(x + y)^2 - 2xy = 4$ va $(x + y)^3 - 3xy(x + y) = 8$ larni yozish mumkin. Bularni e’tiborga olsak, berilgan sistema quyidagi ko‘rinishga keladi:

$$\begin{cases} (x + y)^3 - 3xy(x + y) = 8, \\ (x + y)^2 - 2xy = 4. \end{cases}$$

Agar biz $u = x + y$, $v = xy$ deb olsak, u holda oxirgi sistemadan quyidagi sistemani hosil qilamiz:

$$\begin{cases} u^3 - 3uv = 8, \\ u^2 - 2v = 4. \end{cases}$$

Bu sistemaning ikkinchi tenglamasidan $v = \frac{u^2-4}{2}$ ni topamiz. Buni sistemaning birinchi tenglamasiga qo'yamiz.

$$u^3 - 3uv = 8,\ u^3 - 3u\,\frac{u^2-4}{2} = 8,\ 2u^3 - 3u^3 + 12u = 16, u^3 - 12u + 16 = 0.$$

$u_1 = 2$ oxirgi tenglamaning ildizlaridan biri ekanligi ravshan. Buni tekshirib ko\rish qiyin emas. Demak, $u^3 - 12u + 16$ uchhad u-2 ga bo'linadi. Bo'lishni bajaramiz va $u^3 - 12u + 16 = (u - 2)(u^2 + 2u - 8)$ ga ega bo'lamiz.

Demak, $u^3 - 12u + 16 = 0$ tenglamani $(u - 2)(u^2 + 2u - 8) = 0$ ko'rinishda yozish mumkin. Bundan esa $u - 2 = 0$ va $u^2 + 2u - 8$=0 tenglamalar hosil bo'ladi. Birinchi tenglamadan $u_1 = 2$, ikkinchi tenglamadan $u_2 = -4, u_3 = 2$ larni topamiz. u ning qiymatlarini o'rniga qo'yamiz:

$$v_1 = \frac{u^2-4}{2} = \frac{2^2-4}{2} = 0,\ v_2 = \frac{(-4)^2-4}{2} = \frac{16-4}{2} = 6,$$

Shunday qilib, biz quyidagi sistemalarga ega bo'ldik:

$$\begin{cases} x + y = 2, \\ xy = 0. \end{cases} \text{ Va } \begin{cases} x + y = -4, \\ xy = 6. \end{cases}$$

Bu sistemalardan birinchisi $x_1 = 0, y_1 = 2$ va $x_2 = 2, y_2 = 0$ yechimlarga ega. Ikkinchi sistema esa yechimga ega emas. Demak, berilgan sistema (0; 2) va (2; 0) lardan iborat ikkita yechimga ega ekan.

Shunday qilib, biz ushbu maqolada ikki noma'lumli ikkita tenglamalar sistemasi, ularning turlari va yechish usullarini yetarlicha misollar yordamida bayon qildik. O'quvchilarda bunday sistemalarni yechish malakalarini shakllantirish ularning kelgusida oliy matematika kursida o'qitiladigan va amaliy tatbiqlarga boy bo'lgan *n* noma'lumli *n* ta tenglamalar sistemasini puxta o'zlashtirishlari uchun poydevor bo'ladi. Ushbu maqolani tayyorlash jarayonida quyidagi xulosalarga keldik:

1. Tenglamalar sistemasini amaliy masalalarning matematik modeli sifatida qarash mumkin.
2. Tenglamalar sistemasini yechishda o'rniga qo'yish usuli eng ko'p qo'llaniladigan usuldir.
3. Ko'p hollarda sistemani yechish uning tenglamalaridan birini bir jinsli tenglamaga keltirishdan iborat bo'lishi mumkin.
4. Tenglamalar sistemasini turli usullar bilan yechishga o'rgatish o'quvchilarda ijodiy fikrlash qobiliyatlarini o'stirishga olib keladi.

Adabiyotlar:

1. O'zbekiston Respublikasi Prezidentining "Matematika ta'limi va fanlarini yanada rivojlantirishni davlat tomonidan qo'llab-quvvatlash, shuningdek, O'zbekiston Respublikasi fanlar akademiyasining V.I.Romanovskiy nomidagi matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to'g'risida"gi qarori, 09.07.2019 yildagi PQ 4387-son. [Internet manba]. URL: https://lex.uz/docs/-4409503 (Murojaat etilgan sana: 15.01.2024).

2. "Ta'lim to'g'risida"gi Qonun. [*Интернет манба*]. URL: https://lex.uz / Qonun hujjatlari ma'lumotlar milliy ba'zasi. 24.09.2020-y., 03/20/637/1313-son. (Murojaat etilgan sana: 11.08.22)

3. *Alixonov S*. Matematika o'qitish metodikasi. – Tashkent: O'qituvchi, 2008. – 357 b.

4. *Tojiyev M., Barakayev M., Xurramov A*. Matematika o'qitish metodikasi. – Tashkent: Fan va texnologiya, 2017. – 296 b.

5. *Тўлаганов Т.* Масалалар ечиш бўйича практикум. – Ташкент: Ўқитувчи, 1981. – 104 b.
6. *Симонов А.Я., Бакаев Д.С., Эпельман А.Г.* и др. Система тренировочных задач и упражнений по математике. – Москва: Просвещение, 1991. – 208 с.
7. *Крамор В.С.* Повторяем и систематизируем школьный курс алгебры и начал анализа. – Москва: Просвещение, 1990. – 416 с.
8. *Потапов Н.К., Олехник С.Н., Нестеренко Ю.В.* Конкурсные задачи по математике. – Москва: *АО* "*Столетие*", 1995. – 537 с.
9. *Литвиненко В.Н., Мордкович А.Г.* Практикум по решению математических задач. – Москва: Просвещение, 1984. – 288 с.
10. *Дыбов П.Т.*, *Осколков В.А.* Задачи по математике. – Москва: Оникс, 2006. – 464 с.
11. *Туманов С.И.* Элементарная алгебра. – Москва: Просвещение, 1970. – 864 с.

METHODS OF LEARNING THE SYSTEM OF EQUATIONS IN GENERAL SECONDARY SCHOOLS AND ACADEMIC LYCEUMS

A. Akhlimirzayev[1]

Ilmiy xabarnoma. Seriya: Pedagogik tadqiqotlar – Scientific bulletin. Series: Pedagogical Research. 2024. 2(78). 21-29.
[1]Andijan State University, Andijan, 170100, str., University, 129 (Uzbekistan). E-mail: agsu_info@edu.uz info@tmc.uz

Key words: Equation, system of equations, solving system of equations, substitution method, addition method, Kramer method, graphical method, artificial method.

It is known that the concepts of equations and systems of equations consist of a mathematical model of a number of problems encountered in practice, as well as a number of important concepts of mathematics. That is why general secondary schools and academic lyceums pay great attention to the study of the system of equations. In them, the study of the system of equations begins with the study of the system of two linear equations with two unknowns.

A system of two linear equations with two unknowns is generally written as follows:

$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases} \quad (1)$$

Here a_1, b_1,a_2, b_2, c_1 and c_2 are some real numbers, x and y are unknown numbers. Such a system of equations is solved by: substitution method, addition method, comparison method, graphic method and determinant method. The article provides examples of the application of these methods with solutions.

In practice, there are systems of two unknown equations, one of which is of the first order, and the other is of the second order; both are secondary or higher. The systems in the first case are solved using the substitution method. The systems in the second case are brought to use the replacement method after certain replacements. The article presents examples of systems of both types with solutions.

In addition to the above, general secondary schools and academic lyceums are taught in the mathematics course in the following form

$$\begin{cases} x^2 + y^2 = a \\ xy = b \end{cases}; \quad \begin{cases} x^4 + y^4 = a \\ x + y = b \end{cases}; \quad \begin{cases} x^5 + y^5 = a \\ x + y = b \end{cases};$$

systems too

$$\begin{cases} x + y = a \\ xy = b \end{cases}$$

it is presented and explained. As an example, examples similar to the following system of equations are presented with their solutions:

Example 7. $\begin{cases} x+y=30, \\ xy=209. \end{cases}$ Solve the system .

Solution: If we look at x and y as the free term 209 and the coefficient of the second term is equal to -30 as the roots of the quadratic equation $z^2-30z+209=0$and solve it x =19, y =11 or $x=11$, we find $y=19$. These will consist of the solutions of the given system.

Some systems of equations, for example, systems of the form $\begin{cases} x^2+y^2=a \\ xy=b \end{cases}$ after certain concrete substitutions $\begin{cases} x+y=a \\ xy=b \end{cases}$ is brought to the system in the form of. Here is an example:

Example -8. $\begin{cases} x^2+y^2=10, \\ xy=3. \end{cases}$ Solve the system .

Solution: We multiply both sides of the second equation of the system by 2 and add the term to the first equation.

We have the equation $x^2+2xy+y^2=16$ or $(x+y)^2=16$. From this we find $x+y=4$ and $x+y=-4$. As a result, we have the following two systems:

$$1)\ \begin{cases} x+y=4, \\ xy=3. \end{cases}; \qquad 2)\ \begin{cases} x+y=-4, \\ xy=3. \end{cases}$$

From the first we find $x_1=1,\ y_1=3$ and $x_2=3,\ y_2=1$, and from the second we find $x_3=-1,\ y_3=-3$ and $x_4=-3,\ y_4=-1$. So, the given system has 4 solutions consisting of (1:3), (3:1), (-1:-3), (-3:-1).

Therefore, this article also focuses on solving such systems. At the end of the article, the conclusions about the system of equations and its study are given.

References:

1. *O'zbekiston Respublikasi Prezidentining "Matematika ta'limi va fanlarini yanada rivojlantirishni davlat tomonidan qo'llab-quvvatlash, shuningdek, O'zbekiston Respublikasi fanlar akademiyasining V.I.Romanovskiy nomidagi matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to'g'risida"gi qarori*, 09.07.2019 yildagi PQ 4387-son. [Decision of the President of the Republic of Uzbekistan, PQ No. 4387 of 07.09.2019. State support for the further development of mathematics education and sciences, as well as V.I. of the Academy of Sciences of the Republic of Uzbekistan. On measures to fundamentally improve the activity of the Romanovsky Institute of Mathematics]. [Online]. URL: https://lex.uz/docs/-4409503 (Date of access: 16.01.2024).

2. (2020) *"Ta'lim to'g'risida"gi Qonun* [Law "On Education"]. [Online]. URL: https://lex.uz / Qonun hujjatlari ma'lumotlar milliy ba'zasi, 24.09.2020-y., 03/20/637/1313-son (Date of access: 11.08.22).

3. Alixonov, S. (2008) *Matematika o'qitish metodikasi* [Mathematics teaching methodology]. Tashkent: O'qituvchi.

4. Tojiev, M., Barakaev, M., Xurramov, A., (2017) *Matematika o'qitish metodikasi* [Mathematics teaching methodology]. Tashkent: Fan va texnologiya.

5. To'laganov, T. (1981) *Masalalar yechish bo'yicha praktikum* [Problem solving workshop]. Tashkent: O'qituvchi.

6. Simonov, A.Ya., Bakaev, D.S., Epelman, A.G. (ed.) (1991) *Sistema trenirovochnyh zadach i uprazhnenij po matematike* [System of training problems and exercises in mathematics]. Moscow: Prosveshchenie.

7. Kramor, V.S. (1990) *Povtoryaem i sistematiziruem shkol'nyj kurs algebry i nachal analiza* [We repeat and systematize the school course of algebra and the beginnings of analysis]. Moscow: Prosveshchenie.

8. Potapov, N.K., Olekhnik, S.N., Nesterenko, Yu.V. (1995) Konkursnye zadachi po matematike [Competition problems in mathematics]. Moscow: AO "Stoletie".

9. Litvinenko, V.N., Mordkovich, A.G. (1984) *Praktikum po resheniyu matematicheskih zadach* [Workshop on solving mathematical problems]. Moscow: Prosveshchenie.

10. Dybov, P.T., Oskolkov, V.A. (2006) *Zadachi po matematike* [Math problems]. Moscow: Oniks.

11. Tumanov, S.I. (1970) *Elementarnaya algebra* [Elementary algebra]. Moscow: Prosveshchenie.

Muallif haqida ma'lumot:

AXLIMIRZAYEV Axmadjon – pedagogika fanlari nomzodi, Andijon davlat universiteti mexanika-matematika kafedrasi professori. E-mail: a-ahm@mail.ru