УДК 517.958(072)

IRRATSIONAL TENGLAMALARNI O'RGANISH BO'YICHA BA'ZI BIR MULOHAZALAR

N.Mamadaliyeva, A.Axlimirzayev, B.Abdukadirov, F.Xalikov

Maqola umumiy o'rta ta'lim maktablarida o'qitiladigan va o'quvchilar qiyin o'zlashtiradigan mavzulardan hisoblangan irratsional tenglamalarni o'rganishga bag'ishlangan bo'lib, unda irratsional tenglamalarni yechishda foydalaniladigan formulalar keltirilgan. Bundan tashqari, maqolada irratsional tenglamalar turlarga bo'lingan va har bir turga kiruvchi irratsional tenglamalarni yechish usullari bayon qilingan. Maqola parametr qatnashgan irratsional tenglamalarni o'rganish bilan yakunlangan.

Kalit so'zlar: *irratsional tenglama, arifmetik ildiz, aniqlanish sohasi, teng kuchli tenglama, chet ildiz.*

Статья посвящена изучению иррациональных уравнений, которые считаются одной из трудноусваемыых тем по математике в средней школе. В ней приведены формулы, используемые при решении иррациональных уравнений. Кроме этого рассматриваются деление иррациональных уравнений на виды и способы их решения. а также даны примеры задач с решениями по каждому виду. В конце статьи приведены примеры иррацоналных уравнений с параметрами, которые решаются нестандартными способами.

Ключевые слова: *иррациональные уравнения, арифметический корень, область определения, равносильные уравнения, посторонний корень.*

Ma'lumki, respublikamiz mustaqillikka erishgandan so'ng ta'lim sohasida ham muhim islohotlar amalga oshirildi. Bu islohotlarning asosiy maqsadi respublikamiz oliy o'quv yurtlarida rivojlangan mamlakatlarda tayyorlanayotgan mutaxassislardan mutlaqo qolishmaydigan malakali mutaxassislarni tayyorlashga qaratilgan. Bunday mutaxassislarni tayyorlashda umumiy o'rta ta'lim maktabi, akademik litsey va oily o'quv yurtlarida o'qitiladigan fanlar ichida matematika fani salmoqli o'rinni egallaydi. Chunki bugungi kunda har qanday mutaxassis o'z faoliyatida matematika va matematik usullarga tez-tez murojaat qiladi. Bu esa umumiy o'rta ta'lim maktablari va akademik litseylarda matematikani o'qitishni yanada takomillashtirish kerakligini bildiradi. Bu borada so'nggi yillarda respublikamiz Prezidenti Sh.M.Mirziyoyev tashabbusi bilan bir qator muhim farmon va qarorlar qabul qilindi. Jumladan, 2019-yil 9-iyuldagi PQ-4387 sonli qaror [1] hamda O'zbekiston Respublikasi Prezidenti Sh.M.Mirziyoyevning Oliy Majlisga Murojaatnomasini [2] keltirish mumkin.

Qabul qilingan mazkur hujjatlarda Respublikamiz ta'lim muassasalarida, jumladan, umumiy o'rta ta'lim maktablari va akademik litseylarda o'qitiladigan matematika fani mazmunini zamon talablariga mos keladigan qilib yangilash hamda bu mazmunni o'quvchilarga innovatsion ta'lim texnologiyalarini qo'llagan holda yetkazish kerakligi ta'kidlangan. Shuning uchun ham biz ushbu maqolada umumiy o'rta ta'lim maktablari matematika kursining mazmundor – uslubiy yo'nalishlaridan biri bo'lgan tenglamalar tarkibiga kiruvchi irratsional tenglamalarni o'rganish bo'yicha tajribalarimizni o'rtoqlashamiz.

Irratsional tenglamalar o'quvchilar ancha qiyin o'zlashtiradigan mavzular sirasiga kiradi. Shuning uchun ham bu mavzularni o'rganishga alohida yondashish kerak bo'ladi.

Irratsional tenglamalarni yechishni avvalo arifmetik ildiz tushunchasi, ildizlar ustida amallar, irratsional ifodalar, kasr maxrajini irratsionallikdan qutqarish kabi tushunchalarni takrorlashdan boshlash kerak.

Irratsional tenglamalarni yechishda ko'pincha

$$\left(\sqrt[n]{f(x)}\right)^n = f(x)(n \in N)$$

formuladan foydalaniladi. Agar bu yerda $n = 2k$ bo'lsa, bu formulani qo'llaganda tenglamaning aniqlanish sohasi kengayadi va $f(x) \geq 0$ cheklov e'tiborga olinadi, $n = 2k + 1$ bo'lganda esa $f(x) \geq 0$ cheklash e'tiborga olinmaydi. Shu va yana ba'zi bir sabablar tufayli irratsional tenglamalarni yechishda topilgan qiymatlarni berilgan tenglamaga qo'yib tekshiriladi.

Aksariyat irratsional tenglamalarni yechishda dastlab ularning aniqlanish sohasini topish kerak bo'ladi. Agar tenglamaning aniqlanish sohasi bo'sh to'plam bo'lsa, u holda tenglama yechimga ega bo'lmaydi. Shuning uchun bu tenglamani yechishga harakat qilish foydasiz. Agar tenglamaning aniqlanish sohasi biror chekli yoki cheksiz to'plamdan iborat bo'lsa, u holda u yechimga ega bo'ladi va uni yechib, ildizlari topiladi. Lekin amalda shunday irratsional tenglamalar uchraydiki, ularning aniqlanish sohasini topish ancha murakkab bo'ladi. Bunday hollarda dastlab tenglama yechilib, so'ngra topilgan qiymatlar tekshirib ko'riladi.

Irratsional tenglamalarni yechishda o'quvchilar har doim $\sqrt[2k]{f(x)}$ ifodaning arifmetik ildizi qaralishini, ya'ni $\sqrt[2k]{f(x)} \geq 0$ bo'lishini esda tutishlari kerak. Irratsional tenglamalarni yechishda asosan quyidagi uch usuldan foydalaniladi:

I. Tenglamani har ikkala tomonini bir xil darajaga ko'tarish.
II. Yordamchi noma'lum (belgilash) kiritish.
III. Sun'iy usul.

Quyida irratsional tenglamalarga misollar keltiramiz:

1. $\sqrt{x^3 - x - 1} + \sqrt[4]{2x + 1} = -2$ tenglama yechilsin.

Yechish: Berilgan tenglama yechimga ega emas. Chunki bu tenglamaning chap tomonidagi har bir qo'shiluvchi musbat va ularning yig'indisi manfiy son bo'la olmaydi.

Javob: ∅.

2. $\sqrt{x^2 + x - 6} + \sqrt{2x + 3} = \sqrt{1 - x}$ tenglama yechilsin.

Yechish: Tenglamani aniqlanish sohasini topamiz. U quyidagi sistemaning yechimidan iborat:

$$\begin{cases} x^2 + x - 6 \geq 0, \\ 2x + 3 \geq 0, \\ 1 - x \geq 0. \end{cases} => \begin{cases} (x+3)(x-2) \geq 0, \\ x \geq -\frac{3}{2}, \\ x \leq 1. \end{cases} => \begin{cases} x \leq -3, x \geq 2, \\ x \geq -\frac{3}{2}, \\ x \leq 1. \end{cases} => \emptyset.$$

Demak, tenglamaning aniqlanish sohasi bo'sh to'plamdan iborat. Bu esa berilgan tenglamani yechimga ega emasligini bildiradi.

Javob: ∅.

3. $\sqrt{x + 5} - \sqrt{2x - 3} = \sqrt{4x - 1}$ tenglama yechilsin.

Yechish: Berilgan tenglamani uning har ikkala tomoni musbat bo'ladigan qilib, ya'ni $\sqrt{x + 5} = \sqrt{2x - 3} + \sqrt{4x - 1}$ ko'rinishda yozamiz va aniqlanish sohasini topamiz. U quyidagi sistemaning yechimidan iborat:

$$\begin{cases} x + 5 \geq 0, \\ 2x - 3 \geq 0, \\ 4x - 1 \geq 0. \end{cases} => \begin{cases} x \geq -5, \\ 2x \geq 3, \\ 4x \geq 1. \end{cases} => \begin{cases} x \geq -5, \\ x \geq \frac{3}{2}, \\ x \geq \frac{1}{4}. \end{cases} => \quad x \geq \frac{3}{2}.$$

Oxirgi tenglamaning har ikkala tomonini kvadratga ko'taramiz.

$\left(\sqrt{x + 5}\right)^2 = \left(\sqrt{2x - 3} + \sqrt{4x - 1}\right)^2$, $x + 5 = 2x - 3 + 4x - 1 + +2\sqrt{8x^2 - 14x + 3}$, $2\sqrt{8x^2 - 14x + 3}$

$= 9 - 5x$, $32x^2 - 56x + 12 = 81 - -90x + 25x^2$, $7x^2 + 34x - 69 = 0$, $x_1 = \frac{-17 + \sqrt{772}}{7}$,

$x_2 = \frac{-17 - \sqrt{772}}{7}$.

Bulardan $x \geq \frac{3}{2}$ shartni $x_1 = \frac{-17+\sqrt{772}}{7}$ qanoatlantiradi.

Javob: $\frac{-17+\sqrt{772}}{7}$.

4. $\sqrt[3]{2x - 1} + \sqrt[3]{x - 1} = 1$ tenglama yechilsin.

Yechish: Tenglamani yechish uchun har ikkala tomonini kubga ko'taramiz. Bunda biz $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$ formuladan foydalanamiz.

$\left(\sqrt[3]{2x - 1} + \sqrt[3]{x - 1}\right)^3 = 1^3$, $2x - 1 + x - 1 + 3\sqrt[3]{(2x - 1)(x - 1)}\left(\sqrt[3]{2x - 1} + +\sqrt[3]{x - 1}\right) =$ 1, $3\sqrt[3]{(2x - 1)(x - 1)} = 3 - 3x$, $\sqrt[3]{(2x - 1)(x - 1)} = 1 - x$. Hosil bo'lgan tenglamani yana kubga ko'taramiz:

$\left(\sqrt[3]{(2x - 1)(x - 1)}\right)^3 = (1 - x)^3$, $(2x - 1)(x - 1) = (1 - x)^3$, $(x - 1)[2x - -1 + (x - 1)^2] = 0$, $(x - 1)(2x - 1 + x^2 - 2x + 1) = 0$, $(x - 1) \cdot x^2 = 0$. Bundan esa $x_1 = 0$, $x_2 = 1$ lar kelib chiqadi. Tekshirib ko'rib, bulardan tenglamani faqat $x = 1$ qanoatlantirishini aniqlaymiz.

5. $\sqrt[3]{x + 1} - \sqrt[3]{x - 1} = \sqrt[6]{x^2 - 1}$ tenglama yechilsin.

Yechish: Tenglamani har ikkala tomonini kubga ko'taramiz:

$\left(\sqrt[3]{x + 1} - \sqrt[3]{x - 1}\right)^3 = \left(\sqrt[6]{x^2 - 1}\right)^3$, $x + 1 - x + 1 - 3\sqrt[3]{x^2 - 1}\left(\sqrt[3]{x + 1} - -\sqrt[3]{x - 1}\right) = \sqrt{x^2 - 1}$, $2 -$ $3\sqrt[3]{x^2 - 1} \cdot \sqrt[6]{x^2 - 1} = \sqrt{x^2 - 1}$, $2 - 3\sqrt{x^2 - 1} == \sqrt{x^2 - 1}$, $2\sqrt{x^2 - 1} = 1$, $x^2 - 1 = \frac{1}{4}$, $x^2 = \frac{5}{4}$, $x_{1,2} =$ $\pm\frac{\sqrt{5}}{2}$.

Tekshirib ko'rib, x ning har ikkala qiymati berilgan tenglamani qanoatlantirishiga ishonch hosil qilish mumkin.

Javob: $-\frac{\sqrt{5}}{2}; \frac{\sqrt{5}}{2}$.

6. $\sqrt[4]{1-x}+\sqrt[4]{15+x}=2$ tenglama yechilsin.

Yechish: Berilgan tenglamani yechish uchun uning har ikkala tomonini biron darajaga (bu holda to'rtinchi darajaga) ko'tarish usulidan foydalanish natijasida ancha murakkab tenglama hosil bo'ladi va uni yechish qiyinlashadi. Shuning uchun bu yerda belgilash (o'rniga qo'yish) usulidan foydalanamiz. Agar $\sqrt[4]{1-x}=a$, $\sqrt[4]{15+x}=b$ deb olsak, u holda $a+b=2$, $a^4=1-x$ va $b^4=15+x$ bo'ladi. Agar oxirgi ikkita tenglamani hadma-had qo'shsak, $a^4+b^4=16$ tenglama hosil bo'ladi. Shunday qilib, biz berilgan tenglamaga teng kuchli quyidagi simmetrik sistemaga ega bo'lamiz:

$$\begin{cases} a+b=2 \\ a^4+b^4=16 \end{cases}$$

Bu sistemaning yechimlari $a=0, b=2$ yoki $a=2, b=0$ lardan iborat.

Demak, biz quyidagi ikkita sistemaga ega bo'ldik:

$$\begin{cases} \sqrt[4]{1-x}=0, \\ \sqrt[4]{15+x}=2. \end{cases} ; \qquad \begin{cases} \sqrt[4]{1-x}=2, \\ \sqrt[4]{15+x}=0. \end{cases}$$

Bu sistemalarni yechib, $x_1=1, x_2=-15$ larni topamiz. Bularning har ikkalasi berilgan tenglamani qanoatlantiradi.

Javob: $-15;\ 1$.

7. $x\sqrt{x^2-15}-\sqrt{x}\sqrt[4]{x^2-15}=2$ tenglama yechilsin.

Yechish: $\sqrt{x}\sqrt[4]{x^2-15}=y\ \ (y>0)$ deylik. U holda $x\sqrt{x^2-15}=y^2$ bo'lib, berilgan tenglama $y^2-y-2=0$ ko'rinishga keladi. Uni yechib, $y_1=2$ va $y_2=-1$ larni topamiz. $y>0$ ni e'tiborga olsak, oxirgi tenglamadan $y=2$ ga ega bo'lamiz. Shunday qilib, biz $\sqrt{x}\sqrt[4]{x^2-15}=2, x^2(x^2-15)=2^4$ yoki $x^4-15x^2-16=0$ bikvadrat tenglamani hosil qildik. Undan $x_1^2=16$, $x_2^2=-1$ lar kelib chiqadi. Ularning birinchisidan $x_{1,2}=\pm 4$ ni topamiz. Ikkinchi tenglama esa yechimga ega emas. Tekshirib ko'rib, $x=4$ berilgan tenglamani qanoatlantirishini ko'ramiz.

Javob: 4.

8.$(3x+1)^2=8\sqrt{x}\left(3x-2\sqrt{x}+1\right)$ tenglama yechilsin.

Yechish: Berilgan tenglamada $x>0$ ekanligi ma'lum. Shuning uchun tenglamani har ikkala qismini hadma-had $x\neq 0$ ga bo'lamiz. Natijada

$\frac{(3x+1)^2}{x}=\frac{8\sqrt{x}(3x-2\sqrt{x}+1)}{x}$, $\frac{(3x+1)^2}{x}=\frac{8(3x-2\sqrt{x}+1)}{\sqrt{x}}$, $\frac{(3x+1)^2}{x}=8\cdot\frac{3x+1}{\sqrt{x}}-16$ tenglama hosil bo'ladi. $\frac{3x+1}{\sqrt{x}}=y(y>0,\ \text{chunki}\ x>0)$ deb olsak, u holda $y^2=8y-16$, $y^2-8y+16=0$, $(y-4)^2=0$ tenglama hosil bo'ladi. Undan $y=4$ ni topamiz.

Shunday qilib, $\frac{3x+1}{\sqrt{x}}=4$ yoki $3x-4\sqrt{x}+1=0$ tenglamaga ega bo'ldik. Bu tenglama $\sqrt{x}$ ga nisbatan kvadrat tenglama bo'lib, undan $\sqrt{x}=1$ va $\sqrt{x}=\frac{1}{3}$ larni, ulardan esa $x_1=1$ va $x_2=\frac{1}{9}$ larni topamiz.

Bularning har ikkalasi berilgan tenglamani qanoatlantiradi. Buni tekshirib ko'rish qiyin emas.

Ba'zi bir irratsional tenglamalar ma'lum bir almashtirishlardan so'ng belgilash (o'rniga qo'yish) usuli bilan yechiladi. Quyida bunday tenglamalarga misollar keltiramiz:

9. $\sqrt{\frac{x-5}{x+2}}+\sqrt{\frac{x-4}{x+3}}=\frac{7}{x+2}\sqrt{\frac{x+2}{x+3}}$ tenglama yechilsin.

Yechish: Tenglamaning aniqlanish sohasi $x+2>0, x+3>0, x+5\geq 0$ va $x-4\geq 0$ larning umumiy qismidan, ya'ni $x\geq 5$ dan iborat. Bu shartda tenglamaning chap tomoni musbat. Shuning uchun berilgan tenglamani

$$\frac{\sqrt{x-5}}{\sqrt{x+2}}+\frac{\sqrt{x-4}}{\sqrt{x+3}}=\frac{7}{\sqrt{x+2}\cdot\sqrt{x+3}} \text{ yoki } \sqrt{x^2-2x-15}+\sqrt{x^2-2x-8}=7$$

ko'rinishda yozish mumkin. Hosil bo'lgan tenglamani yechish uchun $x^2-2x--8=y$ belgilash qilamiz. Natijada

$\sqrt{y-7}+\sqrt{y}=7$, $\sqrt{y-7}=7-\sqrt{y}$, $y-7=49-14\sqrt{y}+y\ \ (7\leq y\leq 49)$ tenglama hosil bo'ladi. Bu tenglamadan $14\sqrt{y}=56$ yoki $\sqrt{y}=4$ kelib chiqadi. Bundan esa $y=16$ ni topamiz. Uning bu qiymatini o'rniga qo'ysak, $x^2-2x-8==16$ yoki $x^2-2x-24=0$ kvadrat tenglama hosil bo'lib, undan $x_1=6, x_2=-4$ larni topamiz. Bulardan $x\geq 5$ shartni $x=6$ qanoatlantiradi.

Javob: 6.

10.$(3-x)\sqrt[3]{\frac{3-x}{x-1}}+(x-1)\sqrt[3]{\frac{x-1}{3-x}}=2$ tenglama yechilsin.

Yechish: $\sqrt[3]{\frac{3-x}{x-1}}=y\neq 0$ belgilash qilamiz. U holda $(3-x)y+\frac{x-1}{y}=2$ yoki $(3-x)y^2-2y+x-1=0$ tenglama hosil bo'ladi. Bu tenglamani y ga nisbatan kvadrat tenglama qilib yechamiz:

$$y_{1,2}=\frac{1\pm\sqrt{1-(3-x)(x-1)}}{(3-x)}=\frac{1\pm\sqrt{1-4x+3+x^2}}{3-x}=\frac{1\pm\sqrt{x^2-4x+4}}{3-x}=\frac{1\pm(x-2)}{3-x};\quad y_1=1,$$

$y_2=\frac{x-1}{3-x}$. Agar yuqoridagi belgilashni e'tiborga olsak, berilgan tenglamaga ekvivalent bo'lgan quyidagi 2 ta tenglamaga ega bo'lamiz:

1)$\frac{3-x}{x-1}=1$;2)$\frac{3-x}{x-1}=\left(\frac{x-1}{3-x}\right)^3$

Ularni yechamiz:

1) $\frac{3-x}{x-1}=1,\frac{3-x-x+1}{x-1}=0,\quad \frac{4-2x}{x-1}=0,\quad x=2;$

2) $\frac{3-x}{x-1}=\left(\frac{x-1}{3-x}\right)^3,\frac{3-x}{x-1}=t$ deb olsak, $t=\frac{1}{t^3}$ yoki $t^4-1=0$ tenglama hosil bo'ladi. Undan $t^2-1=0$ $(t^2+1\neq 0)$ ga ega bo'lamiz. U $t_1=1$ va $t_2=-1$ ildizlarga ega. $t=1$ bo'lganda birinchi tenglama hosil bo'lib, undan $x=2$ ni topganmiz. $t=-1$ bo'lsa $\frac{3-x}{x-1}=-1$ bo'lib, bu tenglama yechimga ega emas.

Javob: 2.

Ba'zi bir irratsional tenglamalarni yechishda trigonometrik almashtirishlardan foydalanish qulay bo'ladi.

11.$x+\frac{2x}{\sqrt{2+x^2}}=\sqrt{2}$ tenglama yechilsin.

Yechish: $x>0$ bo'lishi ravshan. $x=\sqrt{2}\operatorname{tg}t$ $(0<t<\frac{\pi}{2})$ almashtirish qilamiz.

$\sqrt{2}\operatorname{tg}t+\frac{2\sqrt{2}\operatorname{tg}t}{\sqrt{2+2\operatorname{tg}^2t}}=\sqrt{2},\ \operatorname{tg}t+\frac{2\operatorname{tg}t}{\sqrt{2(1+\operatorname{tg}^2t)}}=1,\ \operatorname{tg}t+\frac{\sqrt{2}\operatorname{tg}t}{\frac{1}{\cos t}}=1,\quad \frac{\sin t}{\cos t}++\sqrt{2}\sin t=1,\quad \sqrt{2}(\cos t-\sin t)=2\sin t\cos t,\quad \sqrt{2}(\cos t-\sin t)==\sin 2t.$ $0<t<\frac{\pi}{2}$ bo'lganda $\sin 2t>0$ bo'lishi ravshan. Demak, bu oraliqda $\cos t-\sin t>0$ yoki $\operatorname{tg}t<1$ bo'lishi kerak. Bunday shartda oxirgi tenglamani har ikkala tomonini kvadratga ko'tarib unga teng kuchli bo'lgan $2(1-\sin 2t)==\sin^2 2t$ yoki $\sin^2 2t+2\sin 2t-2=0$ tenglamani hosil qilamiz. Bu $\sin 2t$ ga nisbatan kvadrat tenglamadir. Uni yechib, $(\sin 2t)_1=\sqrt{3}-1$ va $(\sin 2t)_2==-\sqrt{3}-1$ larni topamiz. Bulardan $0<\sin 2t<1$ shartni $\sqrt{3}-1$ qanoatlantiradi. Demak, biz $\sin 2t=\sqrt{3}-1$ tenglamaga ega bo'ldik. Agar biz $\sin 2t=\frac{2\operatorname{tg}t}{1+\operatorname{tg}^2t}$ bo'lishini e'tiborga olsak, $\frac{2\operatorname{tg}t}{1+\operatorname{tg}^2t}=\sqrt{3}-1$ tenglamani hosil qilamiz. Bu $\operatorname{tg}t$ ga nisbatan kvadrat tenglama bo'lib, undan $\operatorname{tg}t_1=\frac{1+\sqrt{2\sqrt{3}-3}}{\sqrt{3}-1}$ va $\operatorname{tg}t_2=\frac{1-\sqrt{2\sqrt{3}-3}}{\sqrt{3}-1}$ lar kelib chiqadi. Bulardan $0<\operatorname{tg}t<1$ shartni qanoatlantiradiganini tanlasak, u holda dastlabki tenglamaning yechimi $x=\sqrt{2}\frac{1-\sqrt{2\sqrt{3}-3}}{\sqrt{3}-1}$ dan iborat bo'ladi.

Javob: $\frac{\sqrt{2}}{\sqrt{3}-1}\left(1-\sqrt{2\sqrt{3}-3}\right)$.

Ba'zi bir irratsional tenglamalarda qatnashgan ildizlarning daraja ko'rsatkichlari har xil, ya'ni juft va toq sonlardan iborat bo'lishi mumkin. Bu holda yuqoridagi tenglamalarni yechishda foydalanilgan usullarni qo'llab bo'lmaydi. Bunday tenglamalarni yechishda har biriga alohida yondashiladi. Bunga quyidagi misolni keltirish mumkin:

12. $\sqrt[4]{x-1}+2\sqrt[3]{3x+2}=4+\sqrt{3-x}$ tenglama yechilsin.

Yechish: Bu tenglamani ham an'anaviy (standart) usullar bilan yechib bo'lmaydi. Tanlash usulidan foydalanib (o'rniga qo'yib tekshirib ko'rish), tenglamaning biror ildizini topishga harakat qilamiz. Buning uchun dastlab tenglamani aniqlanish sohasini topamiz. U $x-1\geq 0$ va $3-x\geq 0$ tengsizliklar yechimlarining umumiy qismidan, ya'ni $1\leq x\leq 3$ kesmadan iborat. Demak, yechimni $[1;3]$ kesmadan qidirish kerak. x ning o'rniga $[1;3]$ kesmadan butun qiymatlarni qo'yib ko'rib, $x=2$ berilgan tenglamani qanoatlantirishini ko'ramiz. Haqiqatdan ham,$\sqrt[4]{2-1}+2\sqrt[3]{3\cdot 2+2}=4+\sqrt{3-2},\quad \sqrt[4]{1}+2\sqrt[3]{8}=4+\sqrt{1},\ 1+2\cdot 2=4+1,\ 5=5.$ Endi biz, berilgan tenglamaning $x=2$ dan boshqa ildizlari yo'qligini ko'rsatsak, masala yechilgan bo'ladi. Bu maqsadda biz$f(x)=\sqrt[4]{x-1}+2\sqrt[3]{3x+2}$ va $g(x)=4+\sqrt{3-x}$ funksiyalarni qaraymiz.

$[1;3]$ kesmada $f(x)$ funksiya o'suvchi, $g(x)$ funksiya esa kamayuvchi ekanligi ravshan. Bunday holda $f(x)=g(x)$ tenglama yechimga ega bo'lsa, u yagona bo'lishi ravshan. Demak, $x=2$ berilgan tenglamaning ildizi ekan.

Javob: 2.

Ba'zi bir irratsional tenglamalarni yechishda tengsizliklardan foydalanish ham mumkin. Bunga quyidagi misolni keltiramiz:

13. $\sqrt{x^2-x-1}+\sqrt{1-x-x^2}=x^2+x+2$ tenglama yechilsin.

Yechish: Bu tenglamani yechishda an'anaviy (standart) usullardan foydalanmoqchi bo'lsak, ancha murakkab tenglama hosil bo'lib, uni yechish yanada murakkablashadi. Shuning uchun bu tenglamani yechish uchun Koshi tengsizligi $\left(\frac{a+b}{2}\geq\sqrt{a\cdot b}\right)$ dan foydalanamiz. Unga asosan

$\sqrt{x^2-x-1}\leq\frac{(x^2-x-1)+1}{2}=\frac{x^2-x-1+1}{2}=\frac{x^2-x}{2}$;

$$\sqrt{1-x-x^2}\leq\frac{(1-x-x^2)+1}{2}=\frac{1-x-x^2+1}{2}=\frac{2-x-x^2}{2}.$$

Demak, $\sqrt{x^2-x-1}+\sqrt{1-x-x^2}\leq\frac{x^2-x+2-x-x^2}{2}=\frac{2-2x}{2}=1-x.$

Shunday qilib, berilgan tenglamadan quyidagi tengsizlikni hosil qilamiz:

$x^2+x+2\leq 1-x,\quad x^2+2x+1\leq 0,\quad (x+1)^2\leq 0$. Bu tengsizlikning yechimi $x=-1$ dan iborat. Demak, berilgan tenglama $x=-1$ dan iborat ildizga ega.

Javob: -1.

14.$x\sqrt{1+x}+\sqrt{3-x}=2\sqrt{1+x^2}$ tenglama yechilsin.

Yechish: Bu tenglamani yechish uchun uning har ikkala tomonini darajaga ko'tarish usulidan foydalanish yanada murakkabroq tenglamaga olib keladi. Bu yerda biz Koshi-Bunyakovskiy tengsizligining xususiy xoli bo'lgan

$$a_1a_2+b_1b_2\leq\sqrt{a_1^2+b_1^2}\cdot\sqrt{a_2^2+b_2^2}$$

tengsizlikdan foydalanamiz (ikki vektorning skalyar ko'paytmasi ular uzunliklari ko'paytmasidan katta emas). Unga asosan

$$x\sqrt{1+x}+1\cdot\sqrt{3-x}\leq\sqrt{1+x^2}\cdot\sqrt{(1+x)+(3-x)}=2\sqrt{1+x^2}.$$

Demak, $(x,1)$ va $(\sqrt{1+x},\sqrt{3-x})$ vektorlar kollinear, ya'ni $\frac{x}{\sqrt{1+x}}=\frac{1}{\sqrt{3-x}}$, $x\sqrt{3-x}=\sqrt{1+x},\quad x^2-3x+x+1=0,\quad (x-1)(x^2-2x-1)=0.$

Bularning birinchisidan $x_1=1$ va ikkinchisidan $x_2=1+\sqrt{2}$, $x_3=1-\sqrt{2}$ lar kelib chiqadi. Tekshirib ko'rib, berilgan tenglamani $x_1=1$ va $x_2=1+\sqrt{2}$ lar qanoatlantirishini ko'ramiz. $x_3=1-\sqrt{2}$ esa chet ildizdir.

Javob: $1;\ 1+\sqrt{2}$.

Yana shunday irratsional tenglamalar uchraydiki, ularni yuqorida ko'rib o'tilgan usullar bilan yechib bo'lmaydi. Bunday tenglamaga quyidagi misolni keltirish mumkin.

15. $\frac{\sqrt{x-1}+\sqrt{x+1}+2}{\sqrt{x-1}+\sqrt{x+1}}=\sqrt{x^2-1}$ tenglama yechilsin.

Yechish: Bu tenglamani ham an'anaviy usullar bilan yechib bo'lmaydi. Berilgan tenglamani aniqlanish sohasi $x\geq 1$ dan iborat. Bu esa $\sqrt{x+1}-\sqrt{x-1}\neq\neq 0$ ekanligini bildiradi. Shuning uchun berilgan tenglamani chap tomonini su'rat va maxrajini bu ifodaga ko'paytiramiz va unga teng kuchli bo'lgan quyidagi tenglamani hosil qilamiz:

$\frac{(\sqrt{x-1}+\sqrt{x+1}+2)(\sqrt{x+1}-\sqrt{x-1})}{(\sqrt{x-1}+\sqrt{x+1})(\sqrt{x-1}-\sqrt{x+1})}=\sqrt{x^2-1},\quad \frac{x+1-x+1+2(\sqrt{x+1}-\sqrt{x-1})}{x+1-x+1}=\sqrt{x^2-1},\ \frac{2+2(\sqrt{x+1}-\sqrt{x-1})}{2}=\sqrt{x^2-1},\quad 1+\left(\sqrt{x+1}-\sqrt{x-1}\right)=\sqrt{x^2-1},\quad \sqrt{x+1}--\sqrt{x-1}=\sqrt{x^2-1}-1.$ Bu tenglama $\left(\sqrt{x+1}-\sqrt{x-1}\right)^2=\left(\sqrt{x^2-1}--1\right)^2\ (x+1\geq 0,\ \ x-1\geq 0,\ \ \sqrt{x^2-1}\geq 1)$ tenglamaga teng kuchli. Bu tenglamadan $x^2=2x$ tenglama hosil bo'lib, undan esa $x_1=2,\ x_2=0$ lar kelib chiqadi. $x\geq 1$ bo'lishi kerakligidan $x_2=0$ tenglamani yechimi bo'la olmaydi.

Javob: 2 .

Ba'zi bir irrtasional tenglamalarda noma'lumdan tashqari parametr ham qatnashishi mumkin. Bunday tenglamalarni parametrli irratsional tenglamalar deb ataladi. Bunday tenglamalar undagi parametrning ba'zi qiymatlarida yechimga ega va ba'zi qiymatlarida yechimga ega bo'lmasligi mumkin. Quyida ularga doir bir nechta misollar ko'rib chiqamiz.

16. $\sqrt{a+x}-\sqrt[3]{a+x}=0$ tenglama yechilsin.

Yechish: $\sqrt[6]{a+x}=y$ deylik. U holda $\sqrt{a+x}=y^3$ va $\sqrt[3]{a+x}=y^2$ bo'lib, berilgan tenglamadan $y^3-y^2=0$ yoki $y^2(y-1)=0$ tenglama kelib chiqadi. Undan esa $y_1=0$ va $y_2=1$ larni topamiz. Demak, $\sqrt[6]{a+x}=$

0 va $\sqrt[6]{a+x}=1$ tenglamalarga ega bo'lamiz. Ularning birinchisidan $x_1=-a$, ikkinchisidan $x_2=1-a$ kelib chiqadi.

Javob: $-a, 1-a$.

17. $\sqrt{x-\sqrt{x-a}}=a$ tenglama yechilsin.

Yechish: $\sqrt{x-a}=y$ deb olsak, $\sqrt{x-\sqrt{x-a}}=\sqrt{x-y}$ bo'lib, berilgan tenglamadan quyidagi sistemani hosil qilamiz:

$$\begin{cases}\sqrt{x-a}=y\\ \sqrt{x-y}=a\end{cases}$$

Bu sistemadan unga teng kuchli bo'lgan quyidagi sistemaga kelamiz:

$$\begin{cases}x-a=y^2\\ \quad y\geq 0\\ x-y=a^2\\ \quad a\geq 0\end{cases}$$

Bu sistemaning birinchi tenglamasidan ikkinchi tenglamasini ayiramiz:

$$x-a-x+y=y^2-a^2,\quad y^2-a^2+a-y=0,$$

$$(y-a)(y+a)-(y-a)=0(y-a)(y+a-1)=0,$$

$\left(\sqrt{x-a}-a\right)(\sqrt{x-a}+a-1)=0$. Bundan quyidagi 2 ta

$\sqrt{x-a}-a=0$, $\sqrt{x-a}+a-1=0$ tenglamalar hosil bo'ladi. Ularni yechamiz: $1)\sqrt{x-a}-a=0$, $\sqrt{x-a}=a$, $x-a=a^2$,

$$x_1=a^2+a\ (a\geq 0).$$

$2)\ \sqrt{x-a}+a-1=0$, $\sqrt{x-a}=1-a$, $x-a=1-2a+a^2$, $x_2=a^2-a+1(0\leq a\leq 1)$.

Demak, $a\in[0;1]$ da berilgan tenglama 2 ta $(x_1 \text{va} x_2)$ ildizga ega $a\in(1;+\infty)$ da bitta (x_1) ildizga ega. $a\in(-\infty;0)$da tenglama ildizga ega emas.

18. $\sqrt[3]{(a+x)^2}+4\sqrt[3]{(a-x)^2}=5\sqrt[3]{a^2-x^2}$ tenglama yechilsin.

Yechish: Tenglamaning har ikkala tomonini kubga ko'taramiz.

$$(\sqrt[3]{(a+x)^2}+4\sqrt[3]{(a-x)^2})^3=(5\sqrt[3]{a^2-x^2})^3,$$

$(a+x)^2+64(a-x)^2+12\sqrt[3]{(a^2-x^2)^2}\left(\sqrt[3]{(a+x)^2}++4\sqrt[3]{(a-x)^2}\right)=125(a^2-x^2)$, $\sqrt[3]{(a+x)^2}+$ $+4\sqrt[3]{(a-x)^2}$ ni o'rniga $5\sqrt[3]{a^2-x^2}$ ni qo'yamiz.

$$(a+x)^2+64(a-x)^2+12\sqrt[3]{(a^2-x^2)^2}\cdot 5\sqrt[3]{a^2-x^2}=125(a^2-x^2),$$

$$a^2+2ax+x^2+64a^2-128ax+64x^2+60a^2-60x^2=125a^2-125x^2,$$

$65x^2-63ax=0$, $x_1=0$, $x_2=\frac{63a}{65}$.

Tekshirib ko'rib, har ikkala ildiz tenglamani qanoatlantirishini ko'rish mumkin.

Javob: 0 va $\frac{63a}{65}(-\infty<a<\infty)$.

19. $\sqrt{x-3}+\sqrt{7-x}=a$ tenglama yechilsin.

Yechish: $\sqrt{7-x}=u$, $\sqrt{x-3}=v$ deylik. U holda $7-x=u^2$, $x-3=v^2$ bo'lib, ularni hadma-had qo'shish natijasida $u^2+v^2=4$ ga ega bo'lamiz. Shunday qilib, biz berilgan tenglamaga teng kuchli bo'lgan quyidagi sistemani hosil qilamiz:

$$\begin{cases} u+v=a \\ u^2+v^2=4 \\ u\geq 0, v\geq 0 \end{cases}$$

Sistemaning birinchi tenglamasidan $u=a-v$ ni topib, ikkinchi tenglamaga qo'yamiz.

$$u^2+v^2=4,\ (a-v)^2+v^2=4,\ a^2-2av++v^2+v^2=4,$$

$2v^2-2av+a^2-4=0$. Bu tenglama v ga nisbatan kvadrat tenglamadir. Uni yechamiz:

$$v_{1,2}=\frac{a\pm\sqrt{a^2-2a^2+8}}{2}=\frac{a\pm\sqrt{8-a^2}}{2};\quad v_1=\frac{a-\sqrt{8-a^2}}{2},$$

$v_2=\frac{a+\sqrt{8-a^2}}{2}$. Bularni belgilash qilgan joyga qo'yamiz:

$$u_1=a-v^2=a-\frac{a-\sqrt{8-a^2}}{2}=\frac{a+\sqrt{8-a^2}}{2};$$

$$u_2=a-v_2=a-\frac{a+\sqrt{8-a^2}}{2}=\frac{a-\sqrt{8-a^2}}{2}.$$

Berilgan tenglama haqiqiy ildizlarga ega bo'lish uchun

$$\begin{cases} 8-a^2\geq 0 \\ a-\sqrt{8-a^2}\geq 0 \end{cases}$$

sistema yechimga ega bo'lishi kerak. Uni yechamiz: $\begin{cases} 8-a^2\geq 0 \\ a-\sqrt{8-a^2}\geq 0 \end{cases}$,

$\begin{cases} a^2\leq 0 \\ \sqrt{8-a^2}\leq a \end{cases}$, $\begin{cases} -2\sqrt{2}\leq a\leq 2\sqrt{2} \\ 8-a^2\leq a^2 \end{cases}$,

$$\begin{cases} -2\sqrt{2}\leq a\leq 2\sqrt{2} \\ a^2\geq 4 \end{cases},\qquad \begin{cases} -2\sqrt{2}\leq a\leq 2\sqrt{2} \\ a\leq -2, a\geq 2 \end{cases},\qquad 2\leq a\leq 2\sqrt{2}.$$

$$x-3=v^2\Leftrightarrow x=3+v^2=3+(\frac{a\pm\sqrt{8-a^2}}{2})^2=$$

$$=3+\frac{a^2+8-a^2\pm 2a\sqrt{8-a^2}}{4}=5\pm\frac{a\sqrt{8-a^2}}{2}.$$

Javob: $a\in[2;2\sqrt{2}]$ bo'lsa, $x=5\pm\frac{a\sqrt{8-a^2}}{2}$.

20. $\sqrt{a-\sqrt{a+x}}=x$ tenglama yechilsin.

Yechish: Noma'lumning va a parametrning mumkin bo'lgan qiymatlari $a+x\geq 0$, $a\geq\sqrt{a+x}$ va tenglamaning chap tomoni arifmetik ildiz ekanligidan $x\geq 0$ bo'lishi kerak. Tenglamani har ikkala qismini kvadratga ko'taramiz va $a-x^2=\sqrt{a+x}$ tenglamani hosil qilamiz. Uning har ikkala qismiga $+x$ ni qo'shib, chap tomonini kvadartlar ayirmasi sifatida qarab ko'paytuvchilarga ajratamiz:

$$\mathrm{a}+\mathrm{x}\text{-}\mathrm{x}^2=\sqrt{\mathrm{a}+\mathrm{x}}+\mathrm{x},\ (\sqrt{\mathrm{a}+\mathrm{x}})^2\text{-}\mathrm{x}^2=\sqrt{\mathrm{a}+\mathrm{x}}+\mathrm{x},$$

$$\left(\sqrt{\mathrm{a}+\mathrm{x}}\text{-}\mathrm{x}\right)\left(\sqrt{\mathrm{a}+\mathrm{x}}+\mathrm{x}\right)=\sqrt{\mathrm{a}+\mathrm{x}}+\mathrm{x},$$

$$\left(\sqrt{\mathrm{a}+\mathrm{x}}+\mathrm{x}\right)\left(\sqrt{\mathrm{a}+\mathrm{x}}\text{-}\mathrm{x}\text{-}1\right)=0.$$

$x \geq 0$ va($\sqrt{a+x} \geq 0$) bo'lgani uchun $x + \sqrt{a+x} \geq 0$.

Agar $x + \sqrt{a+x} = 0$ bo'lsa, u holda $x = 0$ va $a + x = 0$. Bu esa $a = 0$ bo'lganda ro'y beradi. Agar $x + \sqrt{a+x} > 0$ bo'lsa, u holda oxirgi tenglikni har ikkala qismini unga bo'lish mumkin. U holda $\sqrt{a+x} = x + 1$ tenglama hosil bo'ladi. Bu tenglamani har ikkala tomonini kvadratga ko'tarib,

$x^2 + x + 1 - a = 0$ tenglamani hosil qilamiz.

Agar $1 - a > 0$ bo'lsa, oxirgi tenglamaning har ikkala ildizi manfiy yoki kompleks sonlar bo'lib, ular berilgan tenglamani yechimi bo'la olmaydi. Agar $a = 1$ bo'lsa, $x_1 = -1$ bo'lib, u ham yechim bo'lmaydi. $x_2 = 0$ berilgan tenglamaning yechimi. Agar $1 - a < 0$ bo'lsa, oxirgi tenglamaning bitta ildizi manfiy, ikkinchisi musbat bo'ladi. Tenglamani musbat ildizini topamiz.

Agar $a > 1$ bo'lsa, $= \frac{-1+\sqrt{-3+4a}}{2}$, agar $a = 1$ bo'lsa, $x = 0$ bo'ladi.

Javob: $x = 0$, agar $a = 0$ bo'lsa, $x = \frac{\sqrt{4a-3}-1}{2}$ agar $a \geq 1$ bo'lsa.

Yuqorida ko'rib o'tilgan misollarni tahlil qilib, irratsional tenglamalarni yechishda asosan uch xil usul: 1) tenglamani har ikkala tomonini bir xil darajaga ko'tarish; 2) belgilash (o'rniga qo'yish) usuli; 3) sun'iy usullardan foydalanish mumkinligini ko'ramiz. Maktab matematika kursida asosan dastlabki ikkita usuldan foydalaniladi. Uchinchi usul bilan yechiladigan tenglamalarni nostandart tenglamalar deb ataladi va bunday tenglamalarni asosan iqtidorli o'quvchilarga tavsiya etiladi. Bunday tenglamalarni yechish jarayonida o'quvchilarning fikrlash qobiliyalari yanada ortadi va pirovard natijada ta'lim jarayoni samarali bo'ladi.

Adabiyotlar

1.O'zbekiston Respublikasi Prezidentining 2019-yil 9-iyuldagi "Matematika ta'limi va fanlarini yanada rivojlantirishni davlat tomonidan qo'llab quvvatlash, shuningdek, O'zbekiston Respublikasi fanlar akademiyasining V.I.Romanovskiy nomidagi matematika instituti faoliyatini tubdan takomillashtirish to'g'risida"gi PQ-4387-sonli qarori. O'zbekiston Respublikasi Qonun hujjatlari ma'lumotlari milliy bazasi, 2019-yil, 07/19/4387/3397-son.

2. O'zbekiston Respublikasi Prezidenti Sh.M.Mirziyoyevning Oliy Majlisga Murojaatnomasi // Yangi O'zbekiston, 2020- yil 25-yanvar.

3. O'zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi "O'zbekiston Respublikasini yanada rivojlantirish bo'yicha Harakatlar strategiyasi to'g'risida"gi PF-4947-sonli farmoni // O'zbekiston Respublikasi qonun hujjatkfhb to'plami. – № 6 (766). 70-модда. – Tashkent: Adolat, 2017.– Б.25 – 150.

4. *Alixonov S.* Matematika o'qitish metodikasi. – Toshkent: Tafakkur-bo'stoni. 2011. – 385 b.

5. *Axlimirzayev A., Xaliqov F., Raximberdiyev E., Ashurov D.*Tenglama va tengsizliklar. – Andijon: "Andijon nashriyot-matbaa" MChJ, 2016. – 219 b.

6. *Axlimirzayev A, Mamadaliyeva N, Xojiyev D, Erkinjonova Sh.* Irratsional tenglama va tengsizliklar. – Toshkent: Innovatsiya-ziyo, 2019. – 128 b.

7. *Василевский А.Б.* Обучение решению задач по математике. – Минск: Высшая школа, 1988. – 255 с.

8. *Крамор В.С.* Повторяем и систематизируем школьный курс алгебры и начал анализа. – Москва: Просвещение, 1990. – 416 с.

9. *Шувалова Э.З., Агафонов Б.Г., Богатырев Г.И.* Повторим математику. – Москва: Высшая школа, 1969. – 464 с.

10. *Колягин Ю.М.* Учись решать задачи. –Москва: Просвещение, 1980. –96 с.

11. *Фридман Л.М., Турецкий Э.Н.* Как научиться решать задачи. – Москва: Просвещение, 1989. – 192 с.

SOME JUDGMENTS ON THE STUDY OF IRRATIONAL EQUATIONS

N.Mamadaliyeva[1], A.Axlimirzayev[2], B.Abdukadirov[3], F.Xalikov[4]

Ilmiy xabarnoma. Seriya. Pedagogik tadqiqotlar – Scientific bulletin. Pedagogical Research. 2020. 2(46). 99–107.
[1,2,3,4]Andijan State University, Andijan, 170100, str. University, 129 (Uzbekistan). E-mail: agsu_info@edu.uz

Keywords: *irrational equations, arithmetic root, domain of definition, equivalent equations, extraneous root.*

It is well known, after gaining independence, important reforms were implemented in the field of education. The main purpose of these reforms is to train highly qualified specialists, who are not inferior to specialists in developed countries. In the training of such specialists, mathematics plays a significant role alongside with the subjects taught at secondary schools, academic lyceums and universities. Today, any specialist often refers to mathematics and

mathematical methods in their work. This means that mathematics education needs to be improved in secondary schools and academic lyceums. In this regard, a number of important decrees and resolutions were adopted by the president of Uzbekistan Shavkat Mirziyoev. PQ of July 9, 2019 "On state support of further development of mathematical education and science, as well as radical improvement of the activity of the Institute of Mathematics of the Academy of Sciences of the Republic of Uzbekistan". Decision No. 4387 and an appeal to the Parliament.

In these documents, the content of mathematics taught in educational institutions of the Republic, including secondary schools and academic lyceums, will be updated to meet modern requirements, and this content will be applied to students using innovative educational technologies. Therefore, in this article, we present experiences in the study of irrational equations, which are part of the equation, which is one of the content-methodological directions of general secondary school mathematics.

The article states that the solution of irrational equations should first begin with a repetition of the notions of arithmetic root, root operations, irrational expressions, and the recurrence of fractions from irrationality. It is argued that the field of acceptable values must be found. In addition, the article presents irrational equations with equations for both types of equations, auxiliary unknown inputs, and solutions of artificial methods and sufficient examples for each species. This article concludes with a study of the irrational equations with which the parametre is included, and presents 5 solutions of the irrational equation with which the parametre is included. Irradiation equations with which the parametre is included are usually included in non-standard equations and use unusual methods for solving them. The study of irrational equations, especially those of irrational equations with a parametre, concluded that students' creative thinking skills are further developed.

References

1.O'zbekiston Respublikasi Prezidentining 2019-yil 9-iyuldagi "Matematika ta'limi va fanlarini yanada rivojlantirishni davlat tomonidan qo'llab quvvatlash, shuningdek, O'zbekiston Respublikasi fanlar akademiyasining V.I.Romanovskiy nomidagi matematika instituti faoliyatini tubdan takomillashtirish to'g'risida"gi PQ-4387-sonli qarori[Presidential Decree of July 9, 2019 "State support for further development of mathematical education and science; Decision No. PP-4387 "On radical improvement of the activity of the Romanovsky Institute of Mathematics"]. O'zbekiston Respublikasi Qonun hujjatlari ma'lumotlari milliy bazasi, 2019-yil, 07/19/4387/3397-son.

2. O'zbekiston Respublikasi Prezidenti Sh.M.Mirziyoyevning Oliy Majlisga Murojaatnomasi (2020) [President of the Republic of Uzbekistan Sh.M.Mirziyoyev's appeal to the Oliy Majlis]. Yangi O'zbekiston. January 25.

3.O'zbekiston Respublikasi Prezidentining 2017 yil 7 fevraldagi "O'zbekiston Respublikasini yanada rivojlantirish bo'yicha Harakatlar strategiyasi to'g'risida"gi PF-4947-son Farmoni (2017) [The Decree of the President of the Republic of Uzbekistan dated February 7, 2017 "On the Strategy of actions for Further Development of the Republic of Uzbekistan" № PD-4947 // The Law of the Republic of Uzbekistan. -№6 (766), Article 70 (766)]. Tashkent. Pp.25-150.

4. Alixonov, S. (2011) *Matematika o'qitish metodikasi* [Methods of teaching mathematics]. Tashkent: Tafakkur-bo'stoni.

5. Axlimirzayev, A., Xaliqov, F., Raximberdiyev, E., Ashurov, D. (2016) Tenglama va tengsizliklar [Equations and inequalities]. Andijan: "Andijon nashriyot-matbaa" MChJ.

6. Axlimirzayev, A., Mamadaliyeva, N., Xojiyev, D., Erkinjonova, Sh. (2019) *Irratsional tenglama va tengsizliklar* [Irrational equations and inequalities]. Tashkent: Innovatsiya-ziyo.

7. Vasilevsky, A.B. (1988) *Obucheniye resheniyu zadach po matematike* [Learning to solve problems in mathematics]. Minsk:Vysshaya shkola.

8. Kramor, V.S.(1990) *Povtoryayem i sistematiziruyem shkolny kurs algebry i nachal analiza* [We repeat and systematize the school course of algebra and the beginnings of analysis]. Moscow: Prosvesheniye.

9. Shuvalova,E.Z., Agafonov,B.G., Bogatyrev,G.I. (1969) *Povtorim matematiku* [Let's repeat the math]. Moscow: Vysshaya shkola.

10. Kolyagin,Yu.M.(1980) *Uchis reshat zadachi* [Learn to solve problems]. Moscow: Prosvesheniye.

11. Fridman,L.M., Turetsky,E.N.(1989) *Kak nauchitsya reshat zadachi* [How to learn to solve problems]. Moscow: Prosvesheniye.

Муллифлар ҳақида маълумот:

Mamadaliyeva Nilufar Baxodirovna – Andijon davlat universiteti maktabgacha ta'lim metodikasi kafedrasi o'qituvchisi. E-mail:mamadaliyeva.nilufar@mail.ru

Axlimirzayev Axmadjon – Andijon davlat universiteti matematika kafedrasi o'qituvchisi. E-mail: a-ahm50@mail.ru

Abdukadirov Boburmirzo Baxodirovich – E-mail: – maktabgacha ta'lim metodikasi kafedrasi o'qituvchisi. E-mail:mr.matematik88@mail.ru

Xalikov Farohiddin Abdumannabovich – Andijon davlat universiteti matematika kafedrasi o'qituvchisi. E-mail: farohiddinxoliqov1974@gmail.com